Panel Caving Scheduling Under Precedence Constraints Considering Mining System

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ABSTRACT

Currently, mine plans are optimised by using several criteria as objective functions, like profit, life-of-mine, concentration of some pollutants, mining costs, confidence level or ore resources, with consideration of constraints related to production rates, plant capacities and grades. Whilst this approach is successful in terms of producing high value production schedules, it uses a static opening sequence of the drawpoints and therefore the optimisation is made within the level of freedom left by the original opening schedule, and as a result it is far from the true optimal value of the project.

This paper presents a model to optimise the sequence of drawpoint opening over a given time horizon. The emphasis is in the precedence constraints that are required to produce meaningful operational sequences considering the exploitation method (panel or block caving), physics considerations and logical rules. Further on, while it applies the standard approach of maximising net present value (NPV), it considers other targets for optimisation, like the robustness and constructability of the plans. Finally, it applies the model to real data to obtain feasible plans by means of this model. The first tests were done varying the production capacity of the mine.

INTRODUCTION

As the industry is faced with more and more marginal reserves, it is becoming imperative to generate mine plans which will provide optimal operating strategies and make the industry more competitive (Chanda, 1990). To obtain these strategies, it is important to consider many constraints, like mining and processing capacity and geomechanical constraints, among others. The construction of the optimisation problems has required rational studies of which mining constraints are applicable in each case (Rubio and Diering, 2004). These constraints are important, because they limit the objective function and define the set of feasible solutions. Obviously, the idea is to get the best solution using an optimisation engine that could search throughout all the feasible solutions that are constrained by the mine design and the geomechanical constraints of a given mining method.

This paper reviews the importance of other variables in underground planning, specifically for the panel caving method. Thus, this paper shows a mathematical model that represents this fact. The model incorporates, the majority of important constraints, including the sequencing (viewed like a set of constraints. The idea is to show the importance of the sequencing in scheduling, and to incorporate an integrated manner to solve the optimisation problem.

As an example that shows what happens if a sequence changes in a fixed 2D model, showing value in \$/t (Figure 1).

In Figure 1, there is a set of blocks (each block could be a drawpoint), with a value. This value is the profit of block. For each period the profit has been calculated, adding block values, and the net present value (NPV) calculated for each case, concluding that the third sequence is the best, this demonstrates the influence sequencing can have on the final result for the objective function, in this example the NPV.

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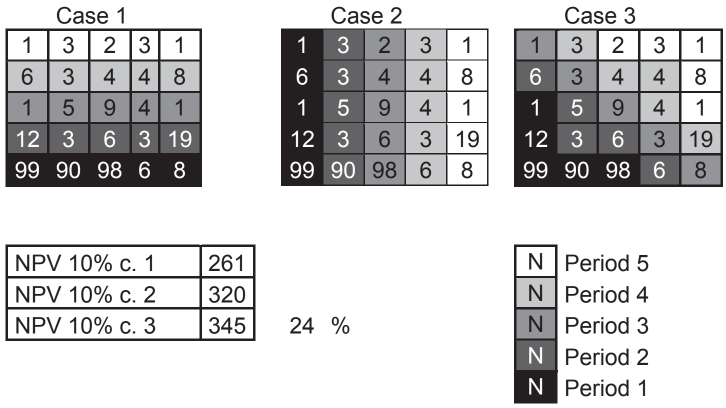

FIG 1 - Alternative sequences for a given two-dimensional fixed value block model.

STATE-OF-THE-ART

A problem close to the one considered in this paper is studied by Chanda (1990), who uses a computerised model for short-term production scheduling, combining simulation with mixed integer programming. He studies the problem of scheduling drawpoints for production, the goal being to reduce as much as possible the fluctuation between periods in the average grade drawn. This model does consider geometric constraints between drawpoints (eg precedence).

Jawed (1993) used another model with the same objective function but focused on operational constraints, manpower requirements, extraction capacity, ventilation requirements, plant capacity and lower bounds on extraction quantity. It was for a room and pillar mine.

Trout (1995) used a model to optimise the mine production schedule. He maximised NPV, and it was applied for sublevel stopping with backfill. He considered sequencing, but the constraints for sequencing of sublevel stopping are very relaxed. For panel or block caving this model is a good start point, but the constraints of precedence and capacities should be modified. Also Carlyle and Eaves (2001) carried out a similar study for the Stillwater Mining Company.

Rahal *et al* (2003) used a mixed integer linear programming for block caving, to solve the optimisation problem. They used as the objective function the deviation from the ideal draw profile. They consider constraints of capacity, precedence, material handling and maximum and minimum levels of drawpoint rates. They showed interesting conclusions. Kuchta, Newman and Topal (2004), present an optimisation model to determine an operationally feasible ore extraction sequence that minimises deviations from planned production quantities. They used aggregation to optimise long-term production planning at an underground mine. The solution applies for a sublevel caving mine (Kiruna) and geometric precedencies were defined in one direction in the horizontal (enough for sublevel caving, but not enough for block caving or sublevel stoping).

Rubio and Diering (2004) solve maximising NPV, for block caving. They use two slices to simulate columns in a discrete vertical model and tested the same objective function as in the Rahal paper. The model uses precedence constraints, defined only for immediate neighbours. Not appear anything about geometrical precedence, considering the time, which the predecessors drawpoints are mined.

Sarin and West-Hansen (2005) solve the planning optimisation problem with mixed integer linear programming. They use NPV an objective function and add penalties for deviations for production and quality. It was developed for room and pillar and longwall mining. The model contains constraints of capacity, sequence (constraints for immediate neighbours), and construction.

Queyranne *et al* (2008) present a model for block caving that maximises the NPV and uses the capacity constraints of mine production, maximum opened and active, drawpoints, neighbour drawpoints. They consider binary variables, and the drawpoints only can be active in a determined number of periods. Also, the constraints of neighbouring drawpoints do not consider a range of time to mine them, but all neighbours are mined in the same period. The model does not have a constraint capacity per drawpoint.

This paper attempts to incorporate sequencing and capacity constraints, locating the sequence in time. For that the sequence needs to be limited geometrically and in this paper demonstrates that. Also it will compare different capacities of mining. The model is based on BOS2 (Vargas *et al*, 2009), that it was a development of Delphos laboratory for open pit mining. The constraints in the model were the capacity of mining and processing, geometallurgy, stocks and geometric constraints. Obviously it was necessary to adapt the model for underground mining, and add some more constraints.

THE OPTIMISATION PROGRAM SCHEDULING

This problem has been studied using different techniques to solve it, but all of these have a common denominator: the constraints. Rubio and Diering in 2004 identifies which constraints should be an optimisation scheduling for block caving, which are presented below:

- Development rate: states the maximum feasible number of drawpoints to be opened at any given time within the schedule horizon, depending on the construction capacity that exists.
- Precedences: defines the order in which the drawpoints will be open. This constraint usually acts on the drawpoint status, activating those that are at the front of the production face. In this model it is a set of two constraints that indicates the maximum number of drawpoints it is possible to advance between two periods in somewhere direction, defining a ratio. Also it defines the number of neighbouring drawpoints that is necessary to mine, in a determined time.
- Maximum opened production area at any given time within the production schedule has to be constrained according to the size of the orebody, available infrastructure and equipment availability.
- Draw rate controls flow of muck at the drawpoint. Draw rate will define the capacity of the drawpoint and it needs to be fast enough to avoid compaction and slow enough to avoid air gaps. In the model presented this constraint will be the production capacity of each drawpoint.
- Draw ratio defines a temporary relationship in tonnage between one drawpoint and its neighbours. It is important to control the dilution. In the model presented, this constraint will be included in the precedencies constraints, by the geometric requirements of panel caving.
- Capacity constraints: forces the mining system to produce the desired production usually keeping it within a range that allows flexibility for potential operational variations. This model uses mine capacity and subunits of the mining system such as cross-cut capacity.

The variables could be integers or real. Normally, the integers variable is used to indicate the state of a point, specifically if the point is opened or not. The real variables are used to specify how much tonnage of drawpoint has been extracted. The logical constraints permits that these two variables are OK.

This model will use the maximisation of NPV in ranking the scenarios. Each point of layout will be evaluated and the idea is to shows the better sequences. Obviously, the objective function could be changed by other.

THE MODEL

The model formulated in the research has been conceptualised for a panel cave mine having several capacity constraints at the production cross-cuts. Also the model integrates the individual value of a drawpoint derived from a premixing algorithm that simulates the vertical flow as well as the economical benefits of withdrawing a drawpoint and its column in a single time period. There are several geometrical constraints that couple the state of neighbours that are present in a panel cave operation. Although the model has been for block caving it is known that this model can be applied to any underground mine.

Variables

The model identifies two variables. The first indicates when the drawpoint is opened. It is a binary variable, that is zero when the drawpoint has not opened and it changes to one, when the drawpoint is opened. The second variable is a real number that represents the percentage of column extracted. It is determined that this variable is accumulated. It defines a set called B that contains all drawpoints, and T, the horizon time. Formally:

$$m_{bt} = \begin{cases} 1 & \textit{if the drawpoint b is opened at } 1\ldots t \\ 0 & \textit{otherwise} \end{cases} \tag{1}$$

$$e_{bt} = \textit{percentage of column extracted of drawpoint b until t}$$
$$e_{bt} \in [0,1],\ b \in B,\ t \in \{1, 2, 3, \ldots, T\} \tag{2}$$

It defines: $\bar{m}_{bt} = m_{bt} - m_{b,t\text{-}1}$, which is equivalent to the values per period, in terms of extraction between t and t - 1. These definitions are based in Equation 1. This variable will be one only in the period of construction of a point. It defines also: $\bar{e}_{bt} = e_{bt} - e_{b,t\text{-}1}$. This variable counts the percentage of column that was extracted between t - 1 and t.

Logical constraints

These constraints establish the basic relations between the variables and mainly state that drawpoints can be opened only once, and that material can be extracted only up to 100 per cent. For each b ∈ B, t = 1, ... ,T - 1:

$$m_{b,t} \leq m_{b,t+1} \tag{3}$$

$$e_{b,t} \leq e_{b,t+1} \tag{4}$$

The first two constraints indicate that, the mining and processing of material of a drawpoint must be done one time. For example: if the block is not removed in t + 1, could not mine in t. In the case that has been mined in t, the constraint forces to mine the point in t + 1.

Production constraints

Production constraints are related to physical and economical limitations of the mine operation, like the maximum amount of tonnage to be extracted per time period, or the minimum amount of material to extract for an opened drawpoint.

The overall mine capacity constraint limits the total amount of mineral to be extracted in the mine for each time period. Considering that each drawpoint has tonnage *ton(r)*, and a upper limit of M^+ tons for the mine, the constraint reads:

$$\sum_{b \in B} ton(b)\bar{e}_{bt} \leq M^+ \quad (\forall t = 1, \ldots, T) \tag{5}$$

Similarly, it is possible constrain the total number of drawpoint to be opened at each time period:

$$\sum_{b \in B} \bar{m}_{bt} \leq P^+ \quad (\forall t = 1, \ldots, T) \tag{6}$$

Notice that these constraints do not consider the ramp up and ramp down, as these are the result of geometric constraints.

Apart from the case above mentioned, the model also considers to replace B by a set $B' \subset B$, to allow for specific constraints on given sets of drawpoints. For example: in the case of a subset of points of cross-cut, it is possible to define an upper bound M_c^+ like the capacity per cross-cut *c* and *C* is a set of cross-cut of layout. In this case the constraint is:

$$\sum_{b \in B_C} ton(b)\bar{e}_{bt} \leq M_c^+ c \in \{1,2,3,\ldots,C\}, \forall t \in \{1,2,3,\ldots,T\} \tag{7}$$

where *Bc* is the set of blocks belonging to cross-cut c. Other constraint considers the limit of capacity per drawpoint. This constraint therefore considers a limit M_b^+: the maximum capacity per drawpoint.

$$ton(b)\bar{e}_{bt} \leq M_b^+ \; (\forall b \in B)(\forall t = 1, \ldots, \mathrm{T}) \tag{8}$$

There is also a constraint limiting the minimum per cent $L \in [0,1]$ to extract from a column if the drawpoint is opened. Notice that this constraint bound the final percentage mined from the column (hence the right side has subindex T).

$$m_{bt}L \leq e_{bT} \quad (\forall b \in B)(\forall t = 1, \ldots, T) \tag{9}$$

Finally, we consider the lifetime of a drawpoint and as upper bound in the number of period it can be operational since it is opened. This is expressed as:

$$\bar{e}_{b,s} \leq \bar{m}_{bt} \quad (\forall b \in B) \ (\forall s = t + A_b, \ldots, T) \tag{10}$$

Geometric constraints

This set of constraints limits the order in which the drawpoints are opened, so this is consistent with technology and geomechanics. The model considers two types of constraints in this category:

1. connectivity constraints, and
2. shape constraints.

To impose these constraints, the model considers a graph whose nodes are the drawpoints. Two drawpoints are connected in the graph if they are close enough (for a certain distance tolerance).

The connectivity constraints force the exploitation to be connected, that is, there are not isolated drawpoints that are opened. This is enforced by considering a set of *access points* from where to start the exploitation is given, so it is possible to calculate a *connected path* $P(b) = (b_1, b_2, b_3, \ldots, b_{k-1}, b_k)$ with $b_k = b$ that goes from the (unique) access point of drawpoint b to Drawpoint b (Figure 2). If we denote $prec(b) = b_{k-1}$ in the path, the connectivity constraint is therefore:

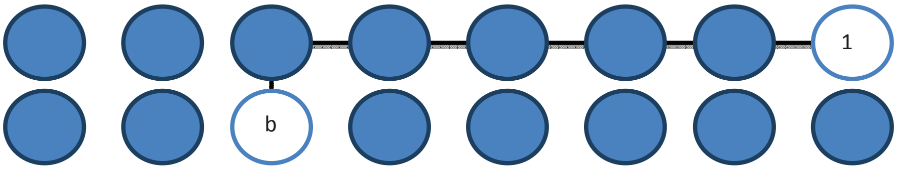

FIG 2 - Example of path from access point to drawpoint.

$$m_{b.t} \leq m_{prec(b),t} \quad (\forall b \in B)(\forall t = 1, \ldots, T) \tag{11}$$

The model considers also two shape constraints. The first one limits the progress of the opened drawpoints on the paths described above: In Figure 3 with red colour is mark that situation. But this constraint not provides the case that in one direction do not mine any block and continue with the others. This case was cover with other constraint that will explain later.

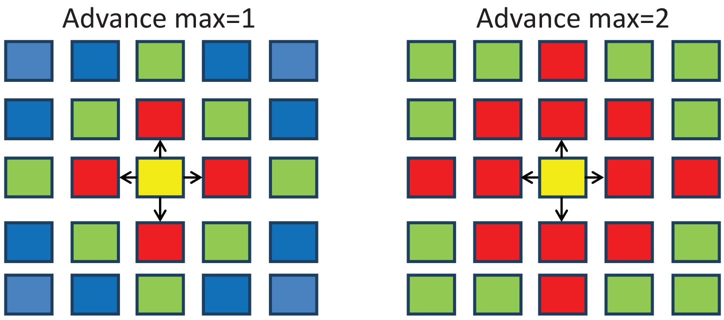

FIG 3 - Example of maximum advance.

$$m_{b+d,t} \leq m_{b,t} \quad (\forall t \leq T) \tag{12}$$

$$m_{h,1} = 0 \ if \ h > d$$

The second type of shape constraints forces that not only drawpoint cannot be opened in an isolated manner, but also that the opening of a drawpoint forces also neighbouring drawpoints to be opened too. Again, in the connectivity graph described before, let $\mathcal{N}(b)$ be the set of drawpoints that are neighbours (connected to) b. This constraint takes also two parameters: K, which is the number of neighbours for a drawpoint to be mined, and Δ, that corresponds to an additional time to do the mining of all the neighbours of some drawpoint. The constraint is:

$$Km_{it} \leq \sum j \in N_N(i) m_{j,t+\Delta} \tag{13}$$

where:

$\#\mathcal{N}_N(i)$ is number of elements of $\mathcal{N}_N(i)$

Figure 4 shows an example of this constraint. Clearly in this example the constraint is fulfil, because only matters the beginning and the end and not necessarily the analysed drawpoint should be the last mined, because the constraint permits that the block i to be mined in t, provided that all the neighbours were mined until t + Δ (before is OK). In the case of example (Figure 4), t = 2 do not have importance, because the constraint is applied on t = 3 and t = 1. Even is possible, that in t = 2 don not happen anything, provided that in t = 3 all the neighbours were mined.

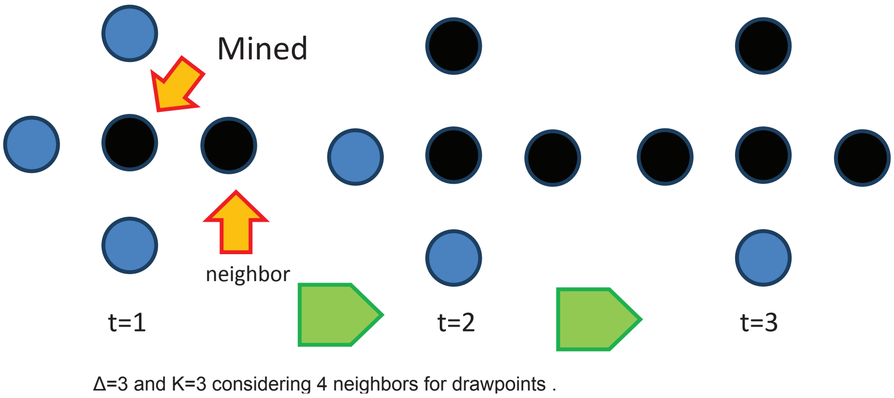

FIG 4 - Example of minimum neighbours.

CASE STUDY

The case study is a mine that has a 30 000 t/d run of mine production. The layout has 332 drawpoints. Actually the sequence was developed, but the idea is to show how varies when capacities per drawpoint or capacities per cross-cut is changed. Production is driven into four crushers located on the orebody footwall (Figure 5). A three-dimensional view of the database is shown.

Obviously all the parameters defined in the last part are need defines.

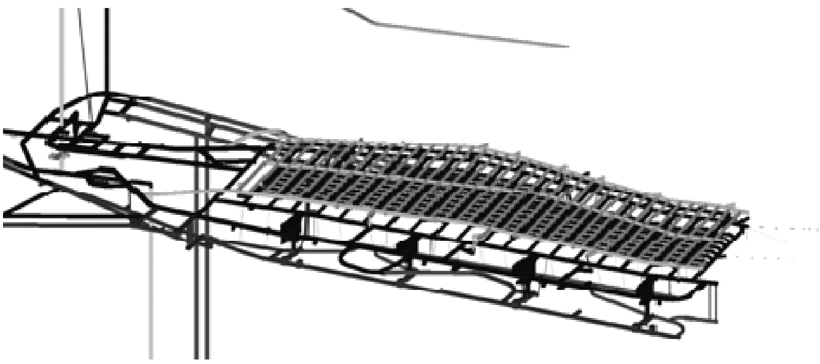

FIG 5 - Case study mining system.

Model inputs

The model has been tested using a 332 drawpoints distributed into 20 production cross-cuts the parameters that remained invariant. These parameters are showed in Table 1.

To begin to study the influence of capacity constraints, the first element of the mining system was the cross-cuts. Also it has considered the capacity per drawpoint. The idea is to run the model with different scenarios and it was selected five to this paper (Table 2), to represent different behaviours. Each case was running considering each drawpoint like a possibility of access.

TABLE 1
Principal static parameters used in the model.

Production capacity	30	Kt/d
Horizon	12	periods
Discount rate NPV	10	%
Maximum advance	3	Dpt/period
Minimum neighbours	6	dpt
Periods to mine neighbours	2	periods
Periods to finish mining of drawpoint	3	Periods
Minimum exploitation column	0.3	%

TABLE 2
Five cases studied varying production.

	Capacity per drawpoint (Kt/yr)	Capacity per cross-cut (t/d)
Case 1	200	5000
Case 2	200	7000
Case 3	200	6000
Case 4	300	6000
Case 5	400	6000

RESULTS

This test was performed with the same 332 drawpoints, that in the last part. All drawpoints were access and after the run all, it was selected the best solution for the objective function (NPV).

Major production capacities got better results. Low capacities of production shows low NPV and with a high dispersion. Lower capacities need more drawpoints to reach the production, so it opens more sideways (Figure 6).

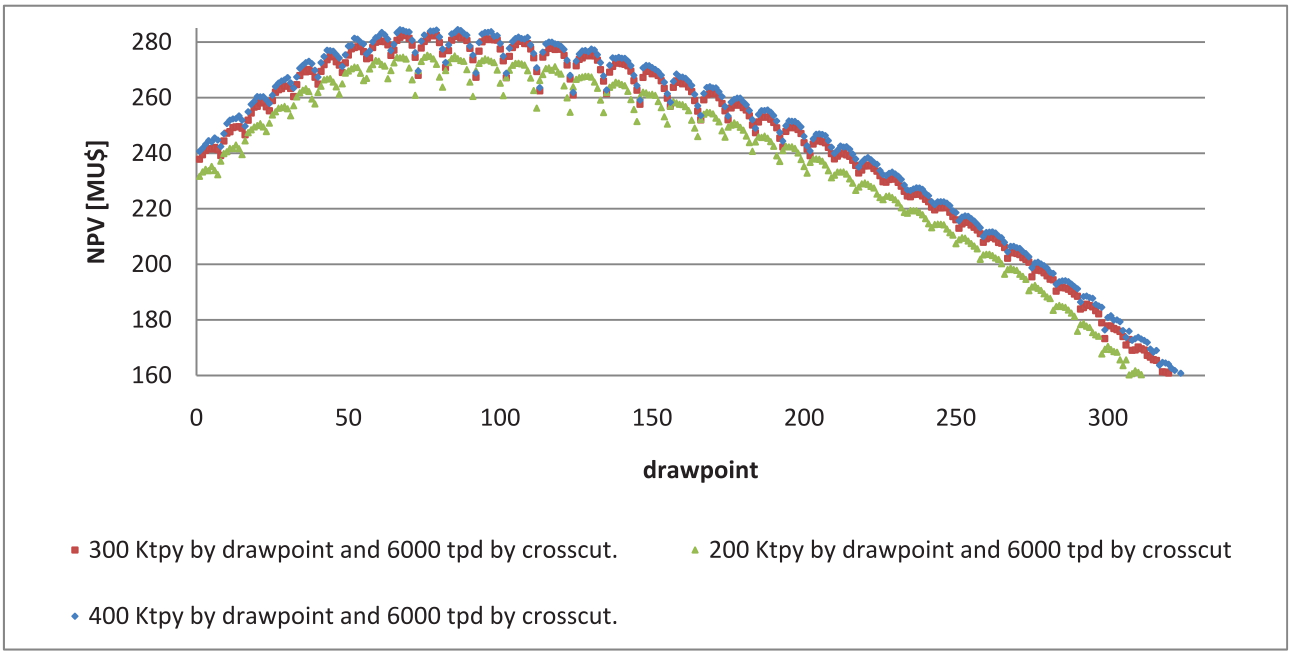

FIG 6 - Net present value considering each drawpoint like access and varying the production per drawpoint.

Other important aspect to review is grade, because objective function was NPV. In the follow graphic It can be seen the difference in the worst grade and the better grade for a same period.

At the ends of the horizon the grades has more dispersion, than in the centre (Figure 7). This is probably because the better grades are not in the centre of the layout. The asymmetry of curves shows that. Obviously this graphic is very valious, because shows clearly, that is not the same to start on a drawpoint or on another.

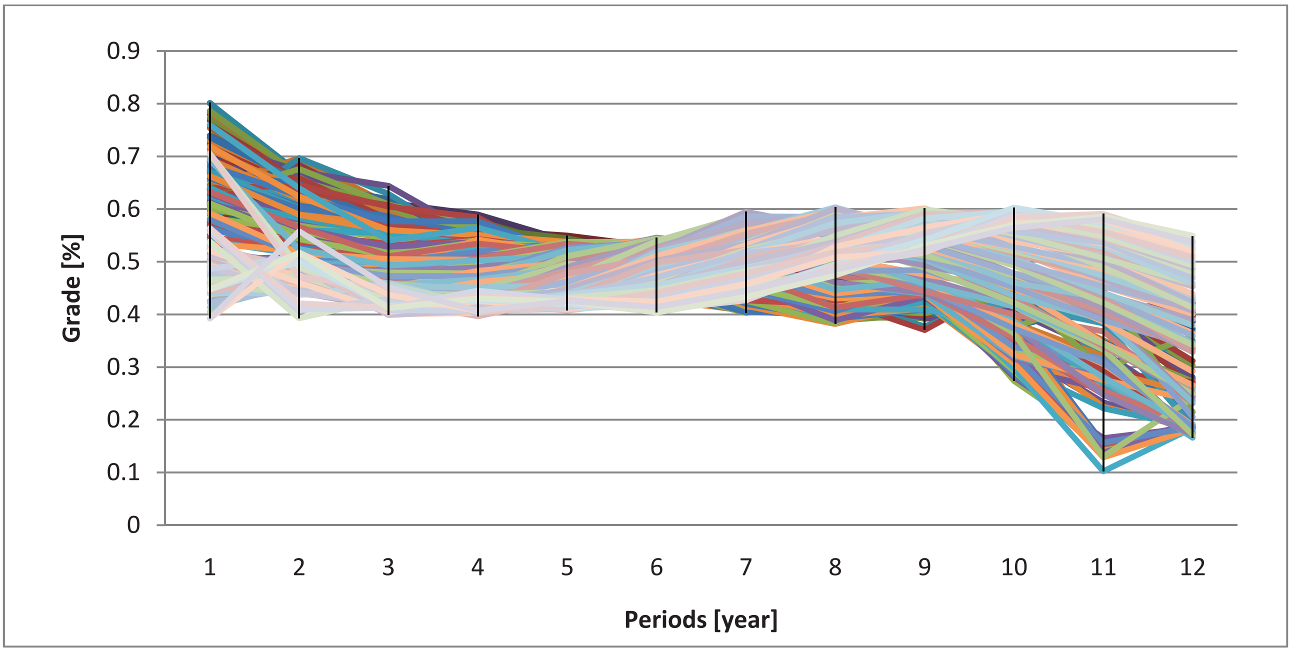

FIG 7 - Behaviour of Cu grade when the access is changed. This is the case with 200 Kt/a per drawpoint and 5000 t/d per cross-cut.

Not important variations could be seen in the grades if it considers a change in a production per cross-cut (Figure 8). The other constraints permit flexibility to these changes in tonnages per cross-cut. It is a good begin to start to incorporate more components of mining system like a crusher.

In reviewing the production plan (Figure 9) it is interesting to note that the plans are similar, but the NPVs are very different. The production capacity constraint was the same for all mining systems, to be evaluated each one, in the same production conditions. For example each rate per cross-cut could mean a different technology to primary crusher, or diameter of shafts. Also the production per drawpoints perfectly could be different technologies of charging material like LHD, Scoop among others. The interesting thing is to evaluate these systems and watch the effects in the objective function, in grades, in mining plans, among others.

The last result to show on this paper is the difference of objective function for the five cases:

Where to start mining is very important, because the difference borders 50 per cent. Could not be attractive for shareholders. The change of technology could be evaluated in a five per cent of the NPV. The idea is to run the model with the complete mining system (Table 3).

Another test was performed was the application of the model shown in this paper for six different mining systems, which differ in the number of crushers for the layout and different number of LHD per cross-cut.

TABLE 3
Results of net present value for each case considered.

	Case 1	Case 2	Case 3	Case 4	Case 5
Min NPV (M US$)	149.2	154.3	152.6	153.6	154.9
Max NPV (M US$)	281.4	282.4	282.3	286.5	287.3
Difference (M U$)	132.17	128.08	129.66	132.89	132.40
Per cent diff (%)	46.97	45.36	45.93	46.38	46.09

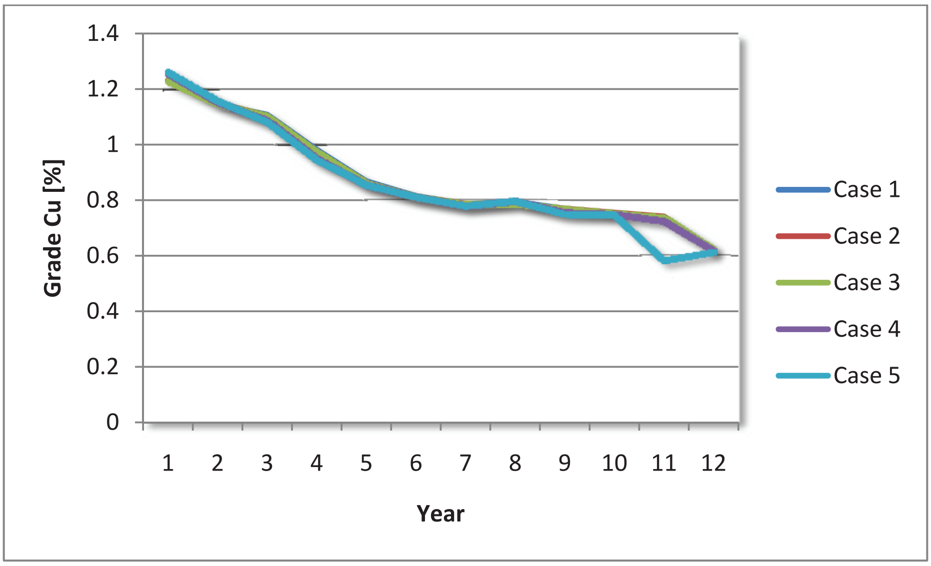

FIG 8 - Grade considering each drawpoint as access. Different production per cross-cut.

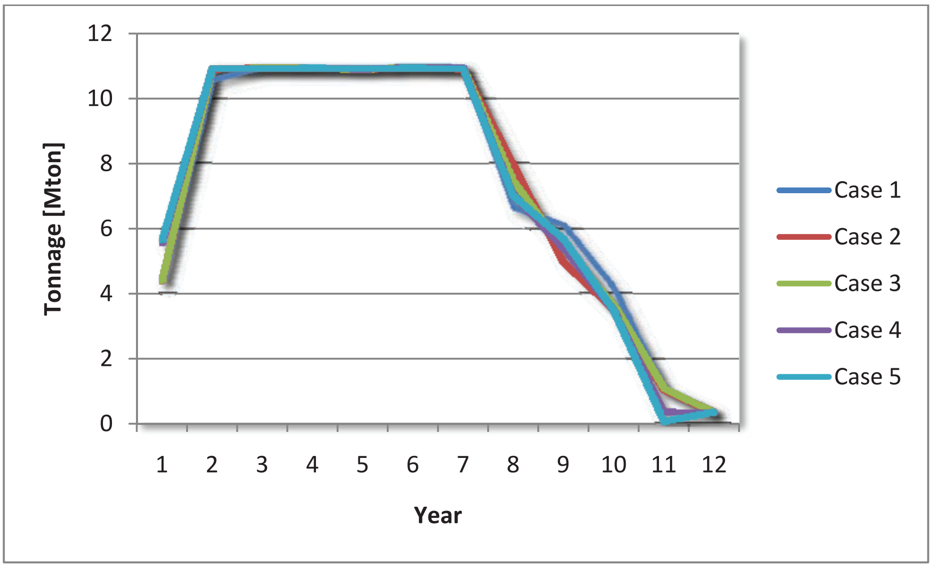

FIG 9 - Grade considering each drawpoint like access, varying the production per drawpoint.

Mining systems considered 20 cross-cuts with a productivity of 3000 t/d by cross-cut, with one or two LHD whose productivity is 3000 t/d to 1500 t/d, respectively. Crushers production capacities was chosen as 7500 t/d. Testing with four, five and six crushers was done to generate six mining systems: the number of crushers increases, production tends to peak in less time, ie the mining plan remains more years in steady state (Figure 10). The grades are similar in the three cases with one LHD per cross-cut. The three cases with two LHD per cross-cut are very similar to the three last cases.

Differences in NPV generated by the different mining systems (Table 4). The best case was the case with six crushers. It is interesting to note that the maximum percentage variation between the best and worst case is around six per cent.

The production by cross-cut could be appreciated in Figure 11, whereas the maximum is 3000 t/d. The rate of each cross-cut corresponds to the peak on the horizon periods. This chart shows cross-cuts with a productivity of between 2000 - 3000 t/d. The maximum capacity is reached in certain periods. It is seen that the upper bound has been somewhat oversized, and there is no uniformity in the extraction rate per cross-cut.

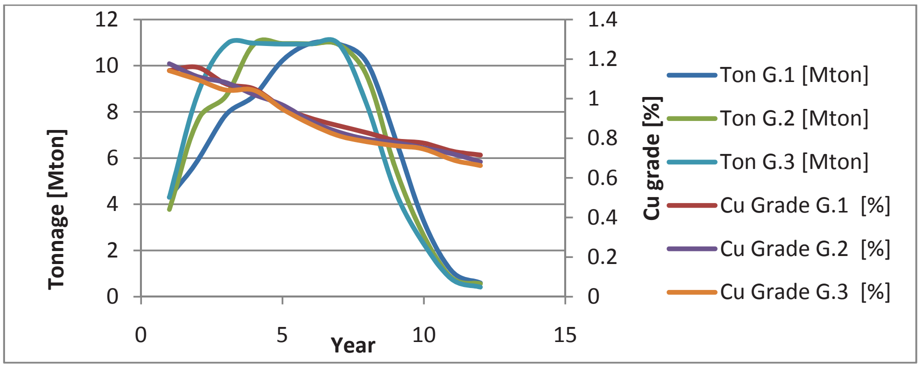

FIG 10 - Production mining plan to the cases with mining systems with four, five and six crushers (one LHD per cross-cut).

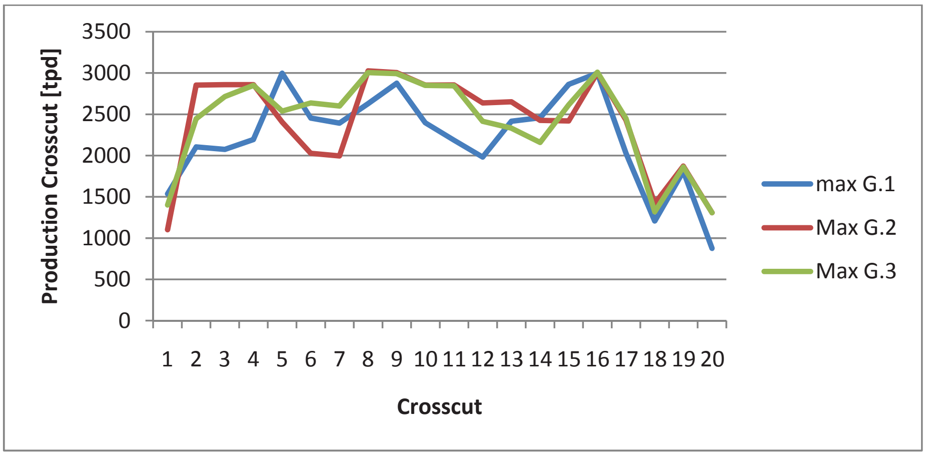

FIG 11 - Grade considering each drawpoint like access, varying the production per drawpoint.

TABLE 4

Net present value results for cases with one and two LHD per cross-cut.

	1 LHD per cross-cut	2 LHD per cross-cut
4 cruchers (M US$)	254.3	252.6
5 crushers (M US$)	263.7	262.0
6 crushers (M US$)	270.5	267.9
Maximum (M US$)	270.5	267.9
Minimum (M US$)	254.3	252.6
Difference (M US$)	16.2	15.3
Per cent (%)	6.00	5.71

In the case of crushers, in most cases, results were obtained with saturated crushers, except the sixth crusher, in the case with six crushers, which had a production of about 4000 t/d. In the case of the cross-cuts, something different happens than with the crushers, as it shows a very irregular behaviour, reaching the 3000 t/d in some cross-cuts. The end of the graph is similar for all three cases.

CONCLUSIONS

Sequence changes the value of objective function significantly. The difference is 50 per cent. The variations in mining plan was because leave free the low limit of capacity. The idea was to show the differences with a sequenced plan and the worst case.

Capacity constraints for different mining system have five per cent of difference in NPV, but it is important that only it has considered the capacity per cross-cut and the capacity per drawpoint. It wills future work to incorporate other components to complete the set of constraints.

By incorporating the capacity of each mining system component as a constraint to the proposed model, there are important NPV differences between different mining systems. However, it is very important to review the productivity obtained for the solution to reach the desired production rate mine.

The main conclusion is that having a proper optimisation engine to assist mine planner to compute the best sequence that suits the strategic objectives of a company is extremely necessary. This initial work has shown that different sequences can show significant differences depending on how the geometries are set up and how the mine design is layout for a given orebody.

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