A comparison of mining cut definition and scheduling algorithms for open-pit short-term mine planning

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Short-term plans must meet the long-term goals while accounting for shovel selectivity, operational space, and scheduling restrictions. There are several approaches that deal with these issues to deliver a feasible short-term schedule. This paper compares two approaches based on mathematical programming to address the short-term mining cut definition and scheduling in an integrated way. They work by scheduling extraction to comply with long-term production goals, but they differ in how mining cuts are modelled. The first approach schedules Selective Mining Units (SMUs) but aggregates them around predefined locations. The second approach does not work on SMUs; it decides directly what mining cuts are to be used from a pre-defined set of shapes. It assigns a destination and schedules the extraction for the selected mining cuts.

We applied both approaches in a real case study to compare their results in terms of profit, mining cut design, expertise needed, profit, and optimisation performance.

INTRODUCTION

Short-term mine planners need to deliver a production plan that attempts to meet the long-term optimum schedule targets and, at the same time, fulfils the mining equipment's operational restrictions such as production rate and selectivity. To achieve this goal, planners rely on updated geological information that is available through blasthole sampling data. Selective Mining Units (SMUs) are defined based on the grade estimation from blasthole data. The block model is updated with a better resolution of the deposit grade.

The next step is defining a feasible short-term schedule using the mining equipment available in each bench. The loading equipment size defines the degree of selectivity achievable in the operation. However, SMUs are usually smaller than the loading equipment selectivity; therefore the process of short-term planning includes the aggregation of SMUs in operational mining cuts, which must be compatible with the selectivity of the loading equipment. Each mining cut is assigned to a single destination, and must fulfil all the operational requirements given by the loading equipment. Short-term planners take these mining cuts as an input to define the short-term operational schedule.

The mining cut definition has an impact on the profit, and the fulfilment of production goals. Long-term planned goals assume perfect selectivity, and accurate estimation of the block model's attributes, but this assumption might not hold in the short-term. Therefore, the planner must define the short-term scheduling, considering real operational constraints, and updated block model data. While this procedure is critical to meeting the production targets, the mining cut definition is usually a trial-and-error, manual process, based mostly on the planner expertise, and several operational rules. There is little support from optimisation tools to guide the planner in this task. This might impact on the ability of the plan to meet the required targets at the operational level.

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In the literature, operational constraints for the short-term schedule are common. However, they have mostly dealt with a mining direction to ensure there is a feasible path from the ramp in each bench and the schedule complies with the extraction advancement (Gholamnejad, 2008; Eivazy and Askari-Nasab, 2012; Yavarzadeh *et al.*, 2014; Mousavi *et al.*, 2016). Favouring the extraction of closer blocks in the same period is also a common approach (Matamoros and Dimitrakopoulos, 2016) to ensure there is enough operational space in each period. However, these approaches fail to address the requirement of assigning the same destination to a given operational area.

To address this shortcoming, different approaches haven been proposed. Dig-limit optimisation is used to define the delimitation between destinations in the same bench accounting for the loading equipment selectivity. Several works have been proposed to deal with this specific issue (Norrena and Deutsch, 2001; Isaaks *et al.*, 2014; Ruiseco *et al.*, 2016; Sari and Kumral, 2018; Vasylchuk and Deutsch, 2019) with different formulations and algorithms to define the optimal separation of materials, while maximising some profit metric or minimising misclassification of individual blocks. These approaches are useful to incorporate selectivity into the destination definition, but they do not include scheduling considerations or a mining cut configuration to define an optimal short-term plan.

Another approach is defining operational mining cuts or polygons. The most prominent works in this matter are related to clustering techniques to group similar blocks. (Tabesh and Askari-Nasab, 2011, 2013) proposed a hierarchical clustering algorithm based on a similarity index. This index incorporated grade, rock type, destination, and distance as similarity metrics. More recently, works incorporated uncertainty in the cluster definition (Tabesh and Askari-Nasab, 2019). As a result, these clustering techniques define an operational mining cut configuration. However, an additional optimisation model (using these clusters as an input) must be used to obtain the short-term plan.

An integrated approach to define a feasible short-term schedule, and mining cut definition simultaneously is usually not addressed in the literature. The objective of this work is to compare two different methods that aim to solve the short-term scheduling, and the mining cut definition problem. The first approach was presented by Nelis and Morales (2021), and it is based on representative SMUs and horizontal precedence to define mining cuts. The second approach is based on direct mining cut assignment from a set of operational shapes. We compare these methodologies in terms of operational feasibility, solution quality, schedule, profit, flexibility, ease of use, and applicability in a real case study.

METHODS

In this section we describe the methodologies compared in this work, and we provide a brief definition of problematic locations in a mining cut definition.

Approach 1: Representative SMUs

The first approach was presented by Nelis and Morales (2021), and it relies on horizontal precedence constraints. Precedence arcs are imposed towards *representative SMUs*. These are special SMUs selected by the mine planner, and they represent the potential location of a mining cut. These representative SMUs are used to impose a destination-based precedence constraint. Every SMU in each bench must be assigned to a unique representative SMU. This assignment is enforced by fulfilling a precedence constraint between the destination of pairs of SMUs: if SMUs are assigned to a representative, they must share the same extraction period and processing facility.

As a result of this assignment, the short-term plan forms connected shapes with the same destination. Other constraints control the size and tonnage of each cut. The shapes generated by the SMU assignment to each representative SMU form the mining cut definition (Figure 1).

The destination-based horizontal precedence arcs can be imposed in a typical short-term scheduling model. Thus, this representative SMU approach can incorporate common scheduling restrictions such as mining and processing capacities or blending and quality targets. Also, horizontal period-based precedence arcs can dictate a required mining direction. This direction ensures a feasible path from the access point in each bench, which is also a common requirement in short-term schedules. The mining cut definition under this approach ensures the fulfilment of these scheduling considerations, and maximises the profit obtained by the destination assignment.

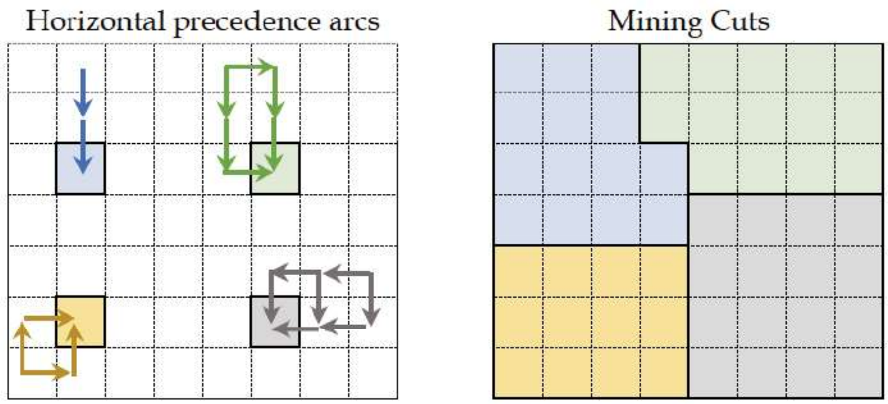

Figure 1. Destination-based precedence arcs directed to different representatives and final mining cuts.

According to the authors (Nelis and Morales, 2021), this approach still presents some challenges. The first one is related to the location of the representative SMUs. The current recommendation is to use a 'best destination' map to identify potential places for mining cuts. Distance between representatives also defines the size of the resulting cuts, and therefore must be considered when defining the representative SMUs. The second challenge is related to the resulting mining cuts' shapes. While the horizontal destination-based arcs ensure connectivity, problematic shapes can still arise in some cases due to the indirect way of imposing operational constraints. This is a common problem in methodologies that define operational shapes, and it is usually addressed by including additional steps to smooth these problematic locations (Tabesh and Askari-Nasab, 2013; Vasylchuk and Deutsch, 2019). The current recommendation by the authors is to fix these shapes in a post-processing stage, using expert criteria.

We will refer to this approach as "representative SMUs" or "first approach" in the rest of the paper.

Approach 2: Direct shape assignment
As a comparison, the second approach relies on explicit enumeration of operational shapes. Instead of imposing the allowed mining cut shape through precedence constraints, feasible shapes are described as an input to the optimisation problem. This enumeration solves the issue of problematic shapes in the resulting cut definition, since the allowable set of shapes only contains operationally feasible definitions.

All SMUs belonging to a given shape must be extracted together, and processed in the same facility to comply with the basic definition of a mining cut. Naturally, each SMU can be assigned to only one shape. There is no overlap between shapes to obtain a feasible mining cut definition.

In this model we can also impose scheduling restrictions such as mining or processing capacities, and blending or quality targets for different attributes. As the number of feasible shapes increases, however, horizontal precedence constraints to control the advancement of the extraction become difficult to incorporate. This is a drawback of this approach compared to the previous one.

Figure 2 shows an example of possible shapes for this approach. The task of defining feasible shapes is made by a mining engineer according to the operational requirements of the loading equipment. The minimum and maximum size of these feasible shapes also respond to operational requirements and scheduling considerations.

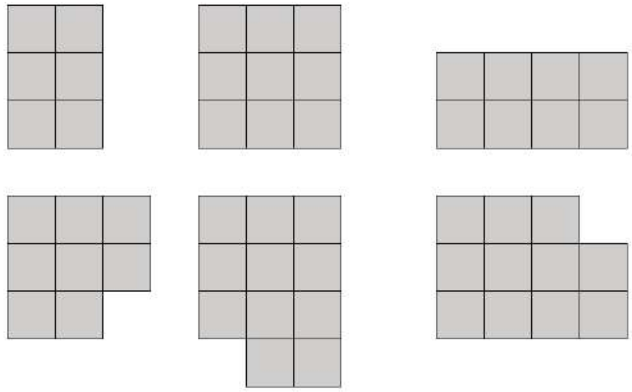

Figure 2. Example of feasible shapes.

We will refer to this approach as "shapes" or "second approach" in the rest of the paper.

EXPERIMENTS

Case Study

To compare both approaches we used the same case study. This case consists of two benches of a copper mine. Copper was estimated using blasthole data with a SMU size of 5x5x10 m^3. Both benches belong to the same mining phase, and we refer them as top (z=95m, 432 SMUs) and bottom (z=85, 352 SMUs) benches. The ramp access point is located at the lower-left bottom on each bench according to the long-term mine design. Figure 3 shows a plan view of the copper grade for each SMU.

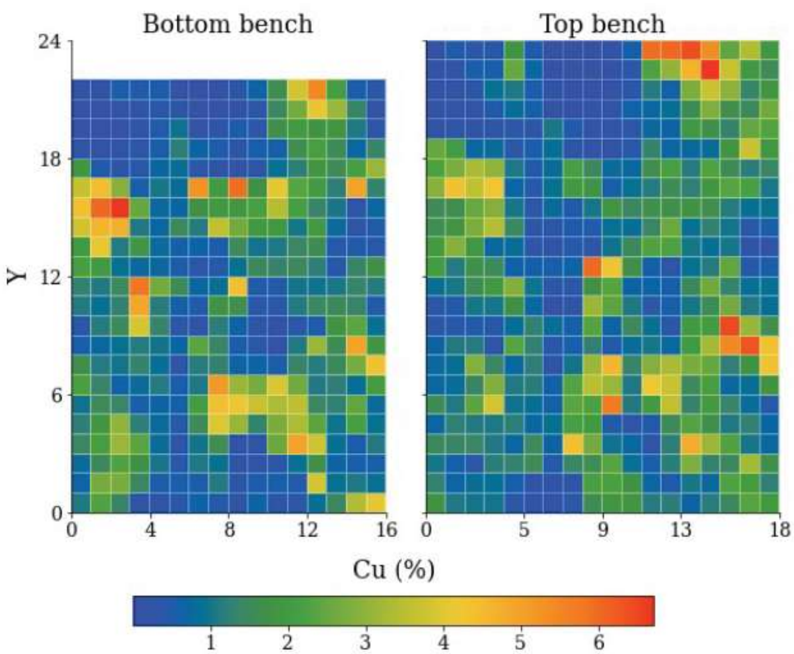

Figure 3. Copper grade for each bench.

Table 1 shows the economic and scheduling parameters used for the case study. We used Equation 1 and 2 to calculate the profit and dump cost for each SMU. We selected the highest-profit destination as the maximum value between the ore profit and waste cost, and fulfilling the processing capacity upper limit.

$$Ore\ Profit = Ton \cdot [(P - C_{sell}) * 22.04 * Cu * R - (C_{min} + C_{proc}) \quad [1]$$

$$Waste\ Cost = -Ton \cdot C_{min} \quad [2]$$

Table 1. Economic and scheduling parameters

Parameter	Value	Units
Copper price (P)	2.0	USD/lb
Mine cost (C_{min})	1.5	USD/ton
Processing cost (C_{proc})	15	USD/ton
Selling cost (C_{sell})	0.5	USD/lb
Metallurgical recovery (R)	60	%
Mine capacity	550,000	ton/week
Processing capacity	224,000	ton/week

Figure 4 shows the selection of representative SMUs for the first approach. We selected the representative SMUs location following the recommendations of Nelis *et al.*, (2021). We attempted to identify zones with a high concentration of ore blocks according to Figure . Operational minimum size for a mining cut was defined as 10 SMUs. For the second approach, we generated a set of 30,305 regular shapes according to the same minimum size of 10 SMUs.

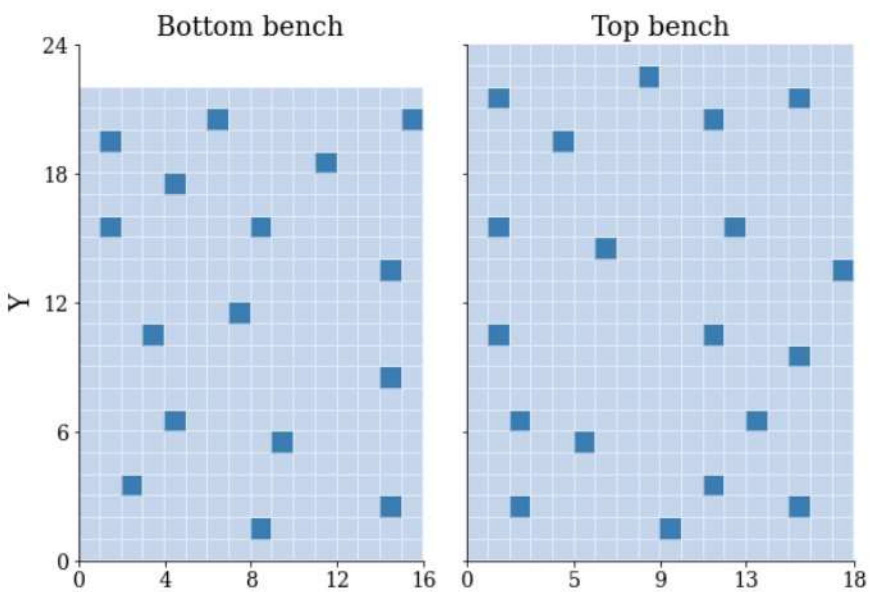

Figure 4. Representative SMUs (shown in dark tone).

Results

The main result of this comparison is the mining cut definition. Figure 5 and Figure 6 show the resulting configuration for both approaches for each bench described in the case study. Colours represent different mining cuts for each approach. For the bottom bench, the first approach used all the potential representative SMUs to define 16 mining cuts. On the other hand, the second approach generated 15 mining cuts. For the top bench, the first approach also used all representative SMUs to define 19 mining cuts, while the second approach used 21 mining cuts.

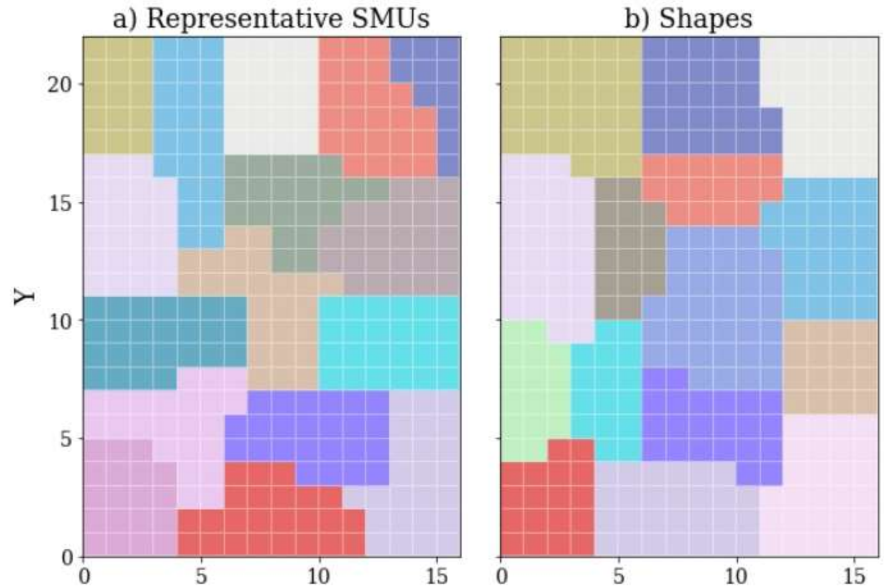

Figure 5. Mining cut definition - bottom bench.

Both approaches defined connected shapes with the required minimum size of 10 SMUs. The connected shapes and required size allow accounting for the shovel selectivity in the extraction process. This is the main objective of the mining cut definition, and both approaches delivered the expected results. However, the shape and location of each cut is different, with no similar patterns between both approaches. At first glance, the first approach generated a more irregular definition of mining cuts (Figures 5a and 6a), compared to the second approach which favoured straight limits between cuts and more regular shapes.

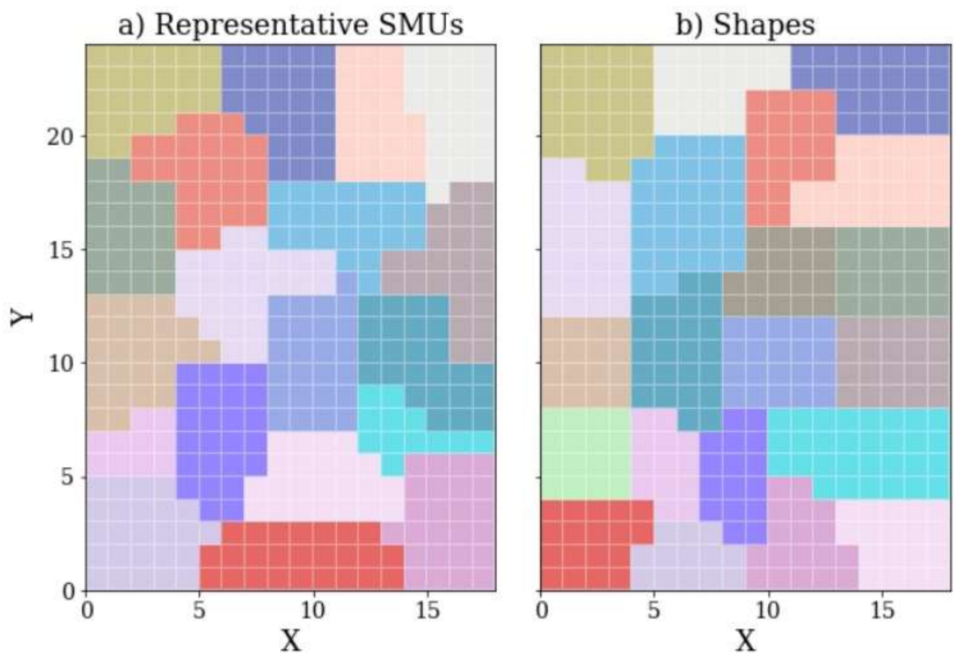

Figure 6. Mining cut definition - top bench.

To further investigate the operational feasibility of the mining cut definitions, we defined a *problematic location* as a SMU assigned to a mining cut which is surrounded mostly by SMUs from other mining cuts. Figure 7 shows several examples of problematic locations derived from a mining cut definition.

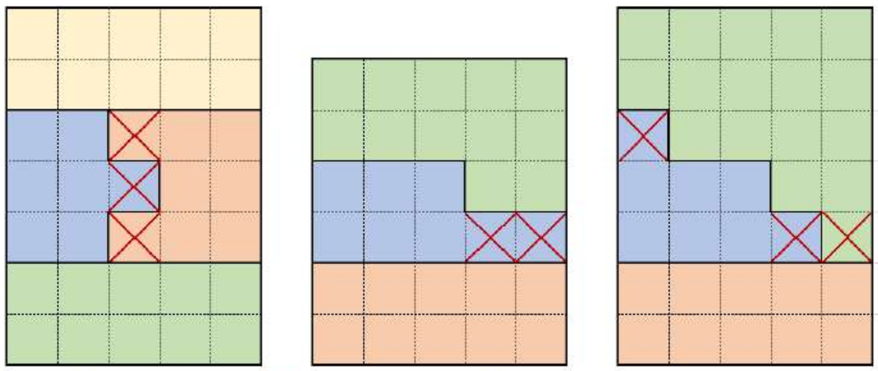

Figure 7. Examples of problematic locations. Colours represent different mining cuts.

The number of problematic locations in the resulting definition is a metric of the operational feasibility of each approach. Figure 8 shows the problematic locations for both approaches according to this definition. This figure highlights the first main difference: the second approach cannot generate problematic locations, because the set of possible shapes only contains feasible configurations.

On the other hand, the first approach presented several problematic locations. The indirect way of defining feasible shapes through precedence constraints is not enough to ensure operational feasibility in the result of the optimisation process. Therefore, this approach based on representative SMUs requires a final feasibility check by a mining engineer.

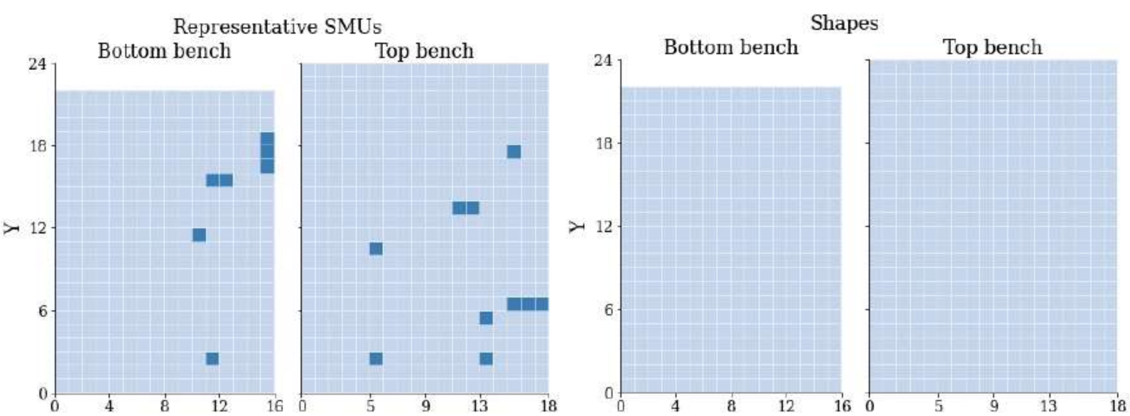

Figure 8. Problematic locations.

Defining the destination of each SMU, and complying with selectivity constraints is often known as dig-limit definition, and there are several other approaches that deal with this specific problem. Both approaches presented in this paper deal with this issue as well. Figure 9 and Figure 10 present the destination assignment for both benches. As a reference, the highest-profit destination for each individual SMU is shown as well (fulfilling capacity constraints).

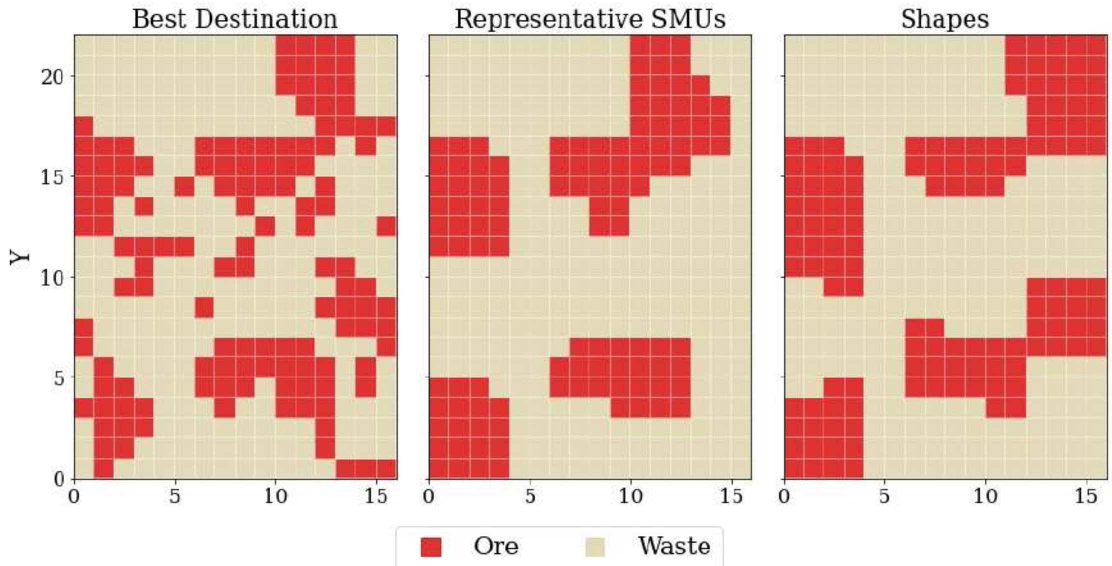

Figure 9. Dig-limit definition - bottom bench.

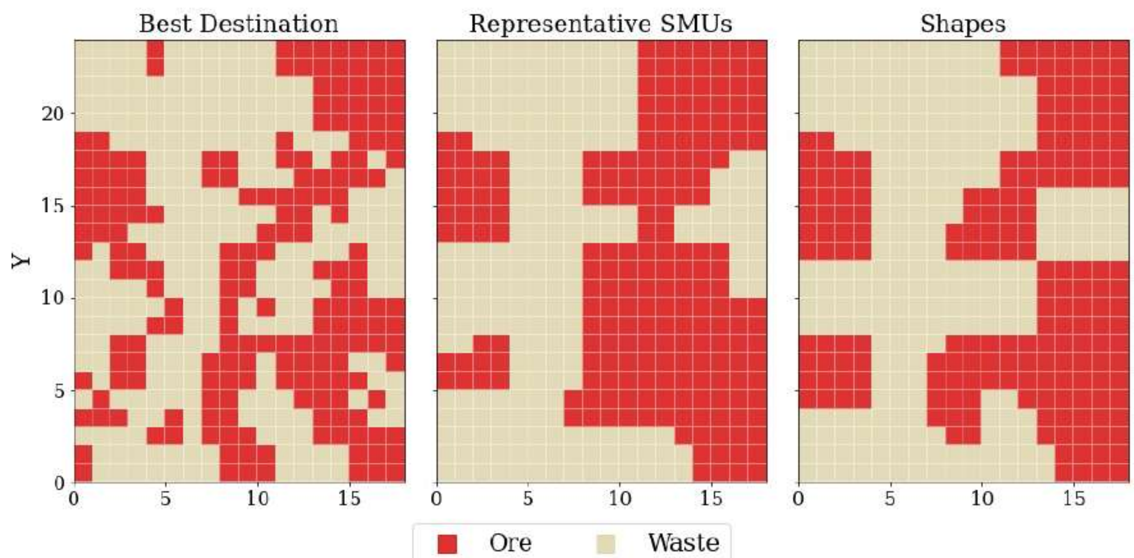

Figure 10. Dig-limit definition - top bench.

As expected, both approaches try to capture the highest-profit destination for each SMU to maximise the total profit of the dig-limit definition. While the mining cut definition was notably different between both approaches, the dig-limits show a more similar destination pattern. In this sense, both approaches work as expected, since they successfully defined clear zones of ore and waste as the dig-limit definition requires. Differences in the definition are due to the distinct ways of imposing operational feasibility, but both results were adequate to separate the materials. Both definitions also fulfil the plant capacity constraint (240,000 ore tons) to comply with the ore processing target.

In terms of profit, short-term definitions are often compared with the highest-profit destination policy which represents the maximum value achievable by perfect selectivity. Depending on the grade distribution in each bench, operational constraints might have a considerable impact on the bench profit. Table 2 shows the profit achieved by each approach compared to the highest-profit destination definition.

Table 2. Profit comparison

Approach	Profit (USD)	Value gap
Best destination	6,483,911	-
Representative SMUs	5,223,225	19.4%
Shapes	5,492,557	15.3%

As expected, both approaches achieve a lower profit compared to the highest-profit destination for each SMU. For this case study, the decrease is significant, with 19.4% for the first approach and 15.3% for the second approach. This decrease is mostly due to the highly erratic grade distribution, with several ore blocks surrounded by waste. With perfect selectivity, all these blocks can be extracted individually. However, with operational considerations, several other waste blocks must be processed too, which could generate an overall negative outcome on the profit.

A common approach in long-term planning is to introduce mining dilution as an estimation of this effect. While this strategy allows for a more realistic long-term decision, the short-term approaches described herein have the potential to incorporate the loading equipment selectivity explicitly in the destination definition, and evaluate the trade-off between dilution and profit.

Both approaches evaluate this trade-off to define the best possible assignment. In this sense, there is a clear difference of almost 5% in favour of the second approach. The classification of each SMU depends directly on the precedence arcs defined towards each representative. A bad decision on their location could be detrimental to the material classification. Therefore, the reliance on representative SMUs in the first approach might result in a loss of profit if they are not chosen carefully. On the other hand, the large set of shapes of the second approach provides more flexibility to capture more profit in this case study.

In terms of optimisation performance, both approaches also differ. Table 3 presents a comparison for the main optimisation results. The approach based on representative SMUs is significantly faster than the second approach. The first approach presents many constraints due to the horizontal precedence arcs directed to each representative SMU. The number of variables, however, remains low due to the restricted block model size used in short-term scheduling. On the other hand, the second approach enumerates all possible mining cuts produced by a set of feasible shapes. Even for small models, the number of possible cuts is significant, and might be prohibitively large for other block models.

Table 3. Optimisation performance

	Representative SMUs	Shapes
Variables	3,904	23,483,508
Constraints	85,489	787
Optimisation time (s)	3.3	1,057.7
Optimality gap	0%	0%

Regarding ease of use, both approaches presented different challenges. The first approach required the definition of representative SMUs, which can impact profitability. This step depends on engineer expertise so it might be problematic in the implementation phase. This highlights the need for a better procedure to define these locations to make this model more approachable for new users. A second challenge is the definition of the precedence arcs. There is not a unique way of defining these arcs towards each representative SMU. Since they define the operational feasibility of the resulting mining cuts, the engineer must choose an adequate way of defining these arcs to comply with the operational requirements.

The second approach presented one main challenge: the description of feasible shapes. The approach requires an explicit definition of an operationally feasible shape. This set must be rich enough to provide enough flexibility in the optimisation process to result in feasible partitions in each bench. Moreover, the profit obtained also depends on the description quality of this set. Currently, the mining cut definition in real operations depends mostly on the engineer's expertise. Therefore, providing an extensive description of operational feasibility might not be an easy task, and there is plenty of room for improvement in this regard for the second approach.

Finally, we compared the ability to incorporate scheduling restrictions in the mining cut definition. The first approach has the flexibility of scheduling several periods, and imposing precedence constraints between mining cuts to obtain a feasible path from the ramp access in each bench. This can be used to obtain a cut definition and mine schedule for longer time horizons with a higher resolution. For example, Figure 11 shows the same case study but scheduled into a five-period horizon. We divided the total capacities into five periods, and incorporated advancement constraints from the ramp access point in the lower left corner of each bench. We also imposed slope precedence constraints to control vertical advancement.

As Figure 11 shows, there is a clear mining direction from the lower corner of each bench which is compatible with the normal extraction process. Moreover, extraction in the bottom bench begins when the top bench is already extracted to comply with the slope precedence restrictions. Also, the bottom bench extraction starts with enough space to allow the shovel operation, which is also desirable in a feasible short-term schedule. This schedule also met the processing target (44,800 tons) in each period to ensure a continuous ore feed. It is worth noting that problematic shapes persist in this schedule. As previously commented, it is a known issue for this approach.

It is not possible to incorporate the same kind of scheduling constraints in the second approach. Imposing precedence constraints between mining cuts is difficult due to the large number of variables involved in the optimisation problem. Calculating all possible arcs between the potential mining cuts is a challenge in itself. Therefore, the second approach could not provide a solution for this scheduling problem.

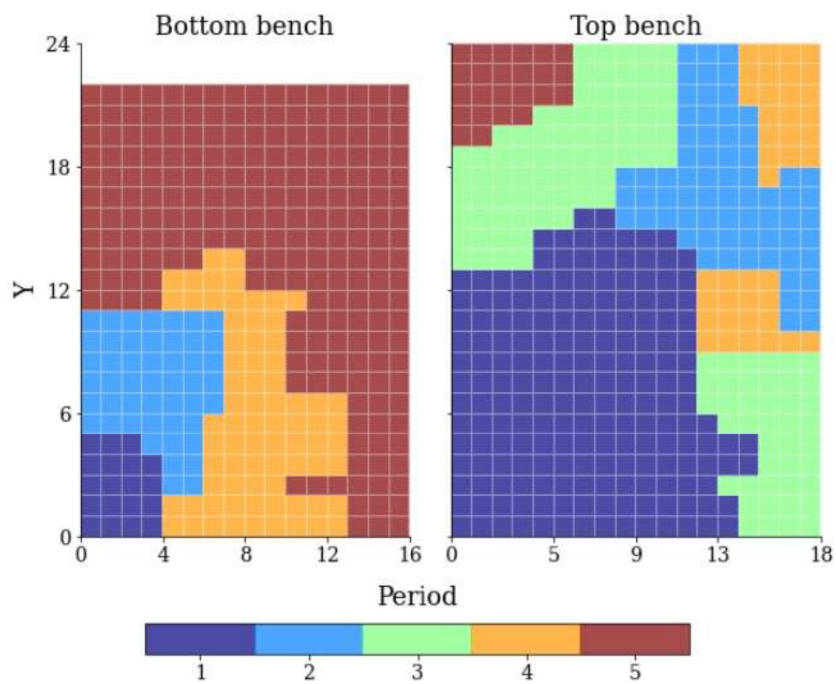

Figure 11. Five-period schedule - Representative SMUs approach.

Summary

Table 4 shows a summary of the comparison between both approaches in the main categories analysed in this study.

Table 4. Comparison summary

	Representative SMUs	**Shapes**
Mining cuts size	Fully controlled	Fully controlled
Mining cut shapes	Connected shapes; indirectly controlled by precedence arcs	Fully controlled
Problematic locations	Yes	No
Dig-limits	Clearly defined, following highest-profit destination	Clearly defined, following highest-profit destination
Profit (USD)	5,223,225	5,492,557
Optimisation Problem	Small; easy to solve	Large; difficult to solve
Scheduling constraints	Yes	No
Challenges	Representative SMUs locations; horizontal precedence definition	Description of feasible mining cuts shapes

CONCLUSIONS

In this paper we presented an application of two different approaches to solve the mining cut definition problem in short-term open-pit mine planning. They both focus on obtaining an operational mining cut definition and maximising profit. Both were able to define connected shapes of adequate size for the loading equipment operation.

The first approach was easier to solve, and could incorporate several scheduling considerations. Its downsides are related to possible problematic locations, and the manual definition of the representative SMUs, which can impact the profit of the schedule. The second approach based on direct shapes assignment presented a higher control over the resulting shapes with no problematic locations by construction. The enumeration of feasible shapes also provides more flexibility to capture higher profit. Its downsides are related to the size of the optimisation problem and the description of a rich set of feasible shapes.

In conclusion, both approaches present advantages and shortcomings. The best approach depends specifically on the priorities set by the short-term mine planner. Future research must deal with the challenges of both approaches to obtain a holistic short-term operational model. A comparison with long-term schedules might be relevant to evaluate the effectiveness of both approaches in meeting strategic production targets such as grade and tonnage when a mining dilution factor is introduced.

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