Comparison of different approaches to strategic open-pit mine planning under geological uncertainty

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Abstract. Uncertainty has been a main topic in mine planning research during the last decade. Several models have been proposed to address and incorporate the uncertainty in the strategic mine planning process, to generate a better, more profitable plan even in adverse conditions such as geological or metal price uncertainty. Among these models, in the general two-stage approach, the extraction is usually the first stage decision while processing considers the uncertainty as a second stage decision. Another option is to minimize deviations from production targets (ore, average grade, contaminants) considering the uncertainty in the grade of the deposit, or the risk-averse approaches where some risk measure is introduced to minimize the losses in unfavorable scenarios. However, it is not clear how these models perform comparatively since they often consider the uncertainty in a fundamentally different way, with different objectives functions, constraints and uncertainty modeling. Therefore, there are no general guidelines under which conditions some approach is better than other. For this reason, this works aims to compare two of these proposed models (two-stage and minimization of deviations) under the same conditions, to verify the advantages of each one of them and generate recommendations about the applicability of these approaches. This comparison is performed considering the production plan, the distribution of the Net Present Value, the total deviations from the target, among other indicators.

Keywords: Strategic Mine Planning, Uncertainty, Stochastic Optimization

1 Introduction

Strategic mine planning must gather and incorporate several sources of information: geology, geomechanical stability, financial, mineral processing, environmental, and others. These factors are often not well defined since they generally involve future behaviors, or their complete characterization is excessively expensive, such as, financial or equipment factors or geological factors, respectively. Therefore, the planner has to rely on estimation of these parameters to obtain the best production schedule possible for the mine operation. There is, however, no guarantee that these estimations will be correct, or even acceptable, when the mine operation is in progress. Bad estimation of some parameters could potentially lead to significant economic losses, which is highly detrimental for the mining project. Moreover, it could lead to wrong investment decision, which are very difficult to modify [1].

The incorporation of uncertainty during the strategic mine planning process has been a major topic of discussion in the last decade towards ensuring that the strategic decisions, such as, mining sequencing, equipment fleet investment and metal production per period consider the uncertainty during the process [2]. Since the deterministic approach in strategic mine planning is based on operational research techniques [3], the incorporation of uncertainty has followed the same approach, using stochastic optimization models to control the uncertainty.

Depending on the type of uncertainty selected to be addressed and how it is modeled, there has been a wide range of approaches to-date. Specifically, for geological uncertainty, the traditional practice has been the use of a single estimate, based on a kriging technique, with the kriging variance as a measure of precision of such estimation. However, it has been shown that, in some cases, the use of a single kriging estimate generates a production plan that is not achievable in the mine operation both in production targets and economical value [1]. More sophisticated techniques, such as, geological simulation, rely on a different paradigm: they produce various possible scenarios, where a single block have a range of possible outcomes showing the local variability seen in real deposits. The use of stochastic techniques allows the mine planner to consider these different geological scenarios during the optimization process to obtain a reliable plan with a good performance for each simulation.

As there is no a unique approach to incorporate the uncertainty in the evaluation, various optimization models have been proposed based on different concepts of the strategic mine planning problem and the impact of the uncertainty. For example, the robust and risk averse approaches focus on obtaining a plan with an acceptable performance in the worst-case scenario to assure a minimum revenue with certain probability or minimizing a risk measure of the schedule [4][5][6]. In the neutral-risk approaches, typically, an expected value over the different scenarios is optimized and no special weight is given to the bad outcomes.

The advantages and disadvantages of these models are not clear and there are no guidelines about which model ought to be used under certain conditions and which produces a higher value, a more reliable plan or other advantage to the mining operation.

This work focuses on comparing the performance of two risk-neutral stochastic mine planning models. The first model is based on the minimization of the deviations from the production targets across every scenario and it was proposed in [7] and [8]. They obtained a single mining sequence that incorporated the uncertainty as a penalty for not meeting the production targets in each geological scenario. This penalty was introduced as a cost in the objective function and the schedule aimed to maximize the expected value of the extraction while minimizing the deviations from the targets. This original formulation was extended to different cases: pushbacks selection under geological uncertainty incorporating penalties [9], mine design optimization based on simulated annealing [10] and joint multi-element uncertainty for an iron deposit [11]. More recently, this approach was applied in mining complexes with multiple processing streams and transportation alternatives with blending constraints using metaheuristics such as simulated annealing and particle swarm optimization to obtain a solution [12] [13]. The results showed that this kind of formulation generates schedules with a lower deviation chance from the actual targets, with larger optimal pit limits and with NPV's up to 25% higher compared to the deterministic optimization techniques.

The second model is a multistage stochastic programming model proposed in [14]. This model defined different decision stages based on the information available about the uncertain parameters. In each stage, the decision made depended on the previous decisions, the information already gathered and the probability distribution for the future outcomes. The multi-stage approach proposed in [14] considered different geological scenarios and incorporated extraction and processing decision that could be modified in each period of the scheduling. The complexity of this model, however, forced the aggregation of blocks and scenarios to obtain a solvable problem. A similar approach was taken in [15] with a two-stage approach, where the first stage was the extraction decision for each period and the second stage decision was the destination of each block which was different for each geological scenario, considering that in the short-term the blasthole information allowed the modification of the processing destination. This model was solved using a modified version of Bienstock -Zuckerberg algorithm [16] and a Toposort heuristic [17]. Another two-stage model was also proposed in [18], but the first and second stage were defined based on the availability of the blasthole information in the short-term to evaluate the effect of gathering this information in advance. These models achieved higher NPV's in comparison to the deterministic optimization, which ranged from 1% up to 10% depending on the case.

2 Methodology

2.1 Minimization of Deviations

As different models can be used to minimize deviations from the production targets, in this paper, the variant found in [19] with a single mine, a single element and without blending constraints was used in the analysis.

Definitions and assumptions. Let B be the set of blocks, R the set of Resources, T the set of periods and S the set of geological scenarios. Let's define $\bar{v}_{bt}$ as the expected value obtained if block $b \in B$ is extracted at period $t \in T$, r_{bs} the resource $r \in R$ of block $b \in B$ considering simulation $s \in S$, and $C_r^{u/l}$ as the upper and lower targets for resource r. The deviation cost from the upper or lower targets for resource $r \in R$ in scenario $s \in S$ is defined as $c_{rs}^{u/l}$ while f^t as the orebody risk discount rate.

Function formulation. The decision variables for this model are:

$$x_{bt} \begin{cases} 1 \text{ if block } b \in B, \text{is extracted at period } t \in T \\ 0 \text{ otherwise} \end{cases} . \quad (1)$$

$$d_{rst}^{u/l} = \text{deviation from target } C_r^{u/l} \text{ in scenario } s \text{ at period } t\,. \quad (2)$$

$$\max \quad \sum_{b\in B}\sum_{t\in T}\bar{v}_{bt}x_{bt} + \sum_{s\in S}\sum_{t\in T}\sum_{r\in R} f^t\, d_{rst}^{u/l}\, c_{rs}^{u/l} \;. \tag{3}$$

$$\text{s.t.} \quad \sum_{b\in B} r_{bs}x_{bt} + d_{rst}^{l} \geq C_r^l \qquad \forall t \in T, s \in S, r \in R\,. \tag{4}$$

$$\sum_{b\in B} r_{bs}x_{bt} - d_{rst}^{u} \leq C_r^u \qquad \forall t \in T, s \in S, r \in R\,. \tag{5}$$

$$\sum_{b\in B} (\beta_{bs} - C_\beta^l)\alpha_{bs}x_{bt} + d_{\beta st}^{l} \geq 0 \qquad \forall t \in T, s \in S, \alpha, \beta \in R\,. \tag{6}$$

$$\sum_{b\in B} (\beta_{bs} - C_\beta^u)\alpha_{bs}x_{bt} - d_{\beta st}^{u} \leq 0 \qquad \forall t \in T, s \in S, \alpha, \beta \in R\,. \tag{7}$$

$$x_{it} \leq \sum_{p=1}^{t} x_{jp} \qquad \forall t \in T, j \in \mathcal{P}(i)\,. \tag{8}$$

$$\sum_{t\in T} x_{bt} \leq 1 \qquad \forall b \in B\,. \tag{9}$$

Equation (3) is the objective function. The first term addresses the maximization of the expected NPV of the extraction, while the second term discounts the deviation costs for every resource considered. The factor f discounts the value from deviations at different periods to introduce a geological risk profile on the schedule. Equations (4) and (5) represent the capacity deviation constraints for upper and lower targets, such as mining and processing limits. Equations (6) and (7) are blending deviations constraints, such as, target metal grade or limits for contaminants. Equation (8) represents the precedence constraint to maintain the order in the extraction, where $\mathcal{P}(i)$ is the set of predecessors for each block i. Finally, equation (9) is the unicity constraint, where each block can be extracted only once.

This formulation is not exactly the same as the one proposed in [19] since the formulation in [19] incorporated dummy constraints to balance the deviation constraints. However, the results are indeed equivalent without those variables and imposed inequalities in the deviation constraints.

2.2 Two-stage stochastic mine planning scheduling

The proposed two-stage stochastic model is based on [15]. The first stage decision considers only the extraction of each block, imposing the same schedule for every geological scenario. The second stage decision selects the best destination for each scenario, aiming to maximize the NPV and fulfil the processing constraints. This two-stage decision framework is similar to the actual mining operation, where the destination decision can be changed in the short term. This model considers that the flexibility to make the long-term scheduling decision to obtain a higher NPV compared to the deterministic scheduling framework.

Definitions and Assumptions. Let B be the set of blocks, R the set of Resources, T the set of periods and S the set of geological scenarios. Let's define $\bar{c}_{bt}$ as the extraction cost of $b \in B$ at period t, r_{bds} the resource $r \in R$ of block $b \in B$ considering simulation $s \in S$ if the block is sent to destination $d \in D$ associated with the second-stage decision, and $\bar{r_c}$ as the resources scenario-independent, associated with the first stage decision. Upper target for resource r is defined as C_r^u and the p_{btds} is the profit obtained if block $b \in B$ is sent to destination $d \in D$ in scenario $s \in S$ at period $t \in T$.

Model formulation. The decision variables of this model are:

$$x_{bt} \begin{cases} 1 \text{ if block } b \in B, \text{is extracted at period } t \in T \\ 0 \text{ otherwise} \end{cases} . \tag{10}$$

$$y_{btds} = \text{fraction of block b sent to destination d at period t in scenario s} . \tag{11}$$

$$\max \quad \sum_{b \in B} \sum_{t \in T} \bar{c}_{bt} x_{bt} + \frac{1}{|S|} \sum_{s \in S} \sum_{t \in T} \sum_{d \in D} p_{btds}\, y_{btds} \,. \tag{12}$$

$$\text{s.t.} \quad \sum_{b \in B} \bar{r}_b x_{bt} \leq C_r^u \qquad \forall t \in T, s \in S, r \in R . \tag{13}$$

$$\sum_{b \in B} \sum_{d \in D} r_{bds}\, y_{btds} \leq C_r^u, \qquad \forall t \in T, s \in S, r \in R . \tag{14}$$

$$x_{bt} = \sum_{d \in D} y_{bdts} \qquad \forall t \in T, s \in S, b \in B . \tag{15}$$

$$x_{it} \leq \sum_{p=1}^{t} x_{jp} \qquad \forall t \in T, j \in \mathcal{P}(i) . \tag{16}$$

$$\sum_{t \in T} x_{bt} \leq 1 \qquad \forall b \in B . \tag{17}$$

Equation (12) is the objective function. The first term represents the cost of extraction while the second term is the expected profit obtained for processing decisions considering every geological scenario. Equation (13) represents the capacity constraints for the extraction, such as mining capacity. Equation (14) represents the capacity constraints for the processing of each block, associated with the second-stage variable. Equation (15) states the relation between x and y variables; a block can be processed only if it was extracted and every fraction of the block was processed. Equation (16) represents the precedence constraints while Equation (17) the unicity constraints.

2.3 Comparison

Both models were implemented for the evaluation of the same deposit to obtain mining schedules under uncertainty. From these schedules, performance indicators were calculated, such as, total ore and waste tonnage. In addition, a comparison between the different extraction decision was performed to evaluate if different approaches would lead to different final pits.

The production plan was compared considering the deviations from the production targets for each scenario and the average ore and waste for each model, to evaluate the mining and processing profile in each period.

An economic analysis was performed aiming to respond how these different methodologies achieved a higher NPV as compare to the traditional case.

3 Results

The study case was a copper porphyry deposit with 14,800 blocks. The scenarios were obtained using sequential gaussian simulation on point support and later a reblock was performed to obtain the final block size. The scheduling and economic parameters for both cases are shown in Tables 1 and 2.

Table 1. Economic Parameters

Mining Cost	1.0	US$/Ton
Processing Cost	10	US$/Ton
Deviation Cost	0.1	US$/Ton
Selling Cost	0.5	US$/lb.
Cu Price	1.5	US$/lb.
Cu Recovery	90	%
Discount Rate	10	%

Table 2. Scheduling Parameters

Periods	5	
Scenarios	10	
Mining Capacity	5.5	MTon/period
Processing Target	4	MTon/period

3.1 Scheduling Results

Table 3 shows a comparison of the value and the final pit for each model, with a deterministic schedule with the same parameters as reference. As expected, both stochastic models achieved a higher NPV compared to the deterministic schedule. The difference between both models was small, both in terms of expected NPV and total tonnage, with the deviations model obtaining a larger and slightly more profitable final pit. Fig. 1 shows a plan view of the schedules. The stochastic models generated a larger pit compared to the deterministic case, but the mining sequence was similar among them.

Table 3. Reserves for each model

	Total Tonnage [MTon]	Reported NPV [US$]	NPV increase
Deviations	27.48	52,912,742	0.81%
Two-Stage	26.48	52,859,761	0.71%

Deterministic	23.85	52,487,241	-

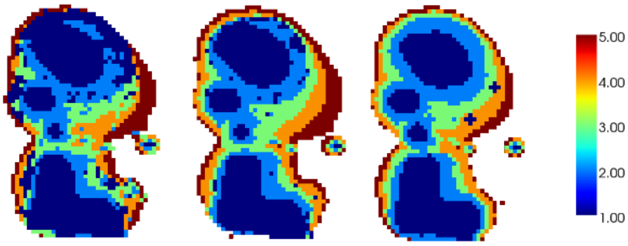

Fig. 1. Plan view of the schedules for different models. From left to right: Deviations, Two-Stage, Deterministic

For a better visualization of the differences between both stochastic models, Fig. 2 shows the final pit limits with a color scale aimed to highlights the sequencing differences: the blue color represents the blocks that are extracted in the same period for both schedules; green color represents the blocks that are extracted in an earlier period in the minimization of deviations model, while the light blue represents blocks that are extracted earlier in the Two-stage model. Orange blocks are extracted only in the minimization of deviations schedule, while dark red blocks are extracted only in the Two-Stage schedule.

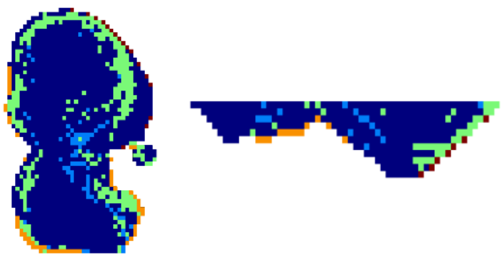

Fig. 2. Comparison of sequences between the stochastic models

The total magnitude of these differences is shown in Table 4, which shows the number of blocks for each category displayed in Fig. 2. The deviations model tended to extract more blocks earlier in the schedule. Also, it can be noticed that both schedules made different final pit decisions, with sets of blocks that are only extracted in one of the two models, which is an indication that both objective functions aimed for a different goal: minimizing the deviations in the first model and taking advantage of the change of destination policy in the second model, as it was detailed in Section 2.

The average production schedules for both models is shown in Fig. 3. The production profile was similar for both models except in period four, where the deviations model shows a higher average ore production. The dispersion of the ore production for both models is similar, with the maximum and minimum ore produced close to the average value, even for the two-stage model, which does not attempt to minimize these deviations explicitly. The average difference between the processing target and the ore scheduled across every period is 884 kton for the deviations model and 913 kton for the two-stage model. Most of this deviation comes from the last two periods, since there is not enough ore to satisfy the production target.

Table 4. Sequence differences between both stochastic models

Category	Number of blocks
Same Period	6618
Earlier in Deviations	949
Earlier in Two-Stage	634
Only in Deviations	441
Only in Two-Stage	125

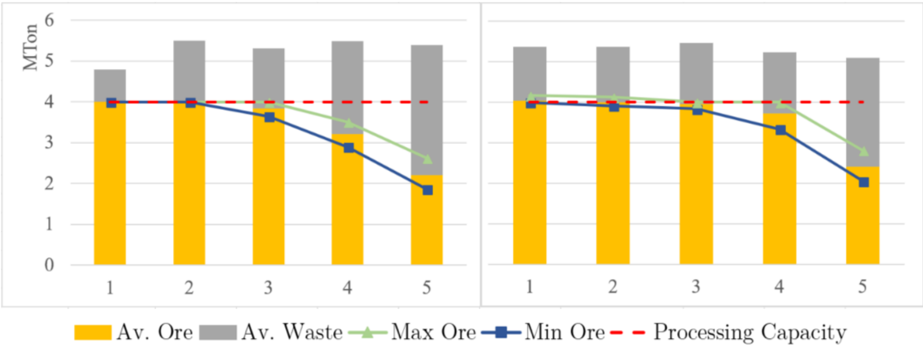

Fig. 3. Production Schedules for Two-Stage model (left) and Deviations model (right)

4 Discussion

The first relevant result is that both models achieve a higher NPV value compared to the deterministic schedule, with larger final pits limits as well, which was an expected result considering the related work. The magnitude of the NPV increase, however, was negligible for both models. The small increase could be related to the fact that this study case is a homogeneous copper porphyry with a single metal of interest. For a similar study case with low uncertainty, a similar result was found in [15] using a two-stage mode, with a negligible NPV increase. For the deviations model, however, a larger NPV increase was expected considering the previous works, where the increases in value ranged between 5% and 25%.

The final pit limits are similar for both models with a 95% of the reserves being common for both models while 80% of the blocks of the final pit are extracted in the same period for both models. Therefore, both models generate a similar pit and sequencing even when the formulations shown in Section 2 are different.

It is relevant, however, noting that the differences between the schedules reflect the different nature of both models. For example, Fig 2, Table 3 and Fig 3 show how the deviation model tends to extract the blocks in earlier periods compared to the two-stage models and how to process more mineral. This behavior is explained by the flexibility introduced by the deviations constraints, since they allow to extract blocks faster if the deviation cost is compensated with a higher revenue. When considering the discount rate of the profit function, processing a block in an early period is more profitable than processing the same block later. With this trade-off, the deviations model produces a more aggressive extraction profile in the first periods to take advantage of the lower discount rate.

While the difference in NPV is negligible between both models, the difference in tonnage is not: the deviation model extracts 5% more ore and 3.8% more tonnage than the two-stage model. This difference is explained by the different formulations. The deviations model aims to extract a higher amount of ore to minimize the deviation cost, while the two-stage model extract less ore but with similar value, since the objective function only considers the maximization of the expected NPV. The higher amount of ore could be beneficial depending of the strategic business model of the operation.

The analysis of the production schedules reveals that the deviation models surpass the processing targets in some scenarios at periods 1 through 4. While a minor excess of ore is manageable, in the short term, a surplus of mineral in every period generates additional handling cost, with an impact on the final NPV. On the other hand, every scenario fulfills the maximum processing capacity in the two-stage model, where the ore target is a hard constraint in the model.

Finally, it is important to mention the selection process of the deviation cost. While the two-stage model does not introduce additional parameters for the schedule, the deviation model requires an additional discount rate and deviation costs for every resource considered. While the literature considers these as control parameters, as a way to introduce the risk profile of the mining engineering on the schedule, the decision of the best deviation cost and discount rate is not trivial. For tonnage deviations, the costs used on previous works range from 2 US$/unit [7] to 10000 US$/unit [19]. For this work, a trial-and-error approach was used, trying to achieve a higher NPV with acceptable deviations, but this selection depends strongly on the study case. A deeper study of the impact of this cost and a recommended methodology to select it is necessary for future works.

5 Conclusion

The stochastic models compared in this work achieved similar NPV values for this study case but emphasizing different extraction strategies. Recommendations of which model is suitable depends on the strategic business plan since the deviations focused on processing more while the two-stage model focused on higher value. Both models achieved a higher value compared to the deterministic case, showing the advantages of stochastic frameworks in strategic mine planning. However, the increase was small in this study case. A comparison of these models in different, more complex orebodies is recommended, to address their differences in a more challenging scenario.

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