Optimal Economic Envelope of Joint Open-Pit and Underground Mines

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ABSTRACT: The costs and complexity of the mine operation increase as extraction in an open pit mine progresses, making the use of underground methods more and more appealing. A natural question is, therefore, what are the boundaries of the open pit and the underground sections of the mine that maximize the economic value of the mining operation? A few attempts have been made to answer this question, ranging from simple approaches that consider scheduling the open-pit section and the underground section sequentially, to more complex approaches that simultaneously schedule both sections at different aggregation levels. The main focus of these works has been determining the optimal transition time. In this work, we focus more on the definition of the economic envelope of the mines rather than time. Our motivation for this is the robustness of this decision. We present a mathematical model and an algorithm to jointly determine the optimal envelops of the open pit and underground sections of the mine.

INTRODUCTION

The selection of the correct extraction method is crucial to optimize the value of a mine project. A deposit can be mined completely as an open pit, and it can also be mined completely underground. There are also deposits whose geometry allows a combined extraction method. The transition from open pit to underground is mainly motivated by the need to reach ore that is deep in the ore deposit, but the ore worth mining is too deep to extract economically (Wang et al., 2012). Financial considerations are not the only factor; the viability of the transition also depends on geological conditions since the rock characteristics affect the stability of the pit slopes and the minimum thickness of the crown pillar (Chen et al. 2003, Bakhtavar et al., 2008). If the mining costs and the geological conditions make the transition feasible, the next step is to choose the appropriate underground method. In this paper, we consider that the objective of the transition is to extend the life of the mine; that is, to continue producing economically. Therefore, an appropriate underground method would be block/panel caving (Hamrin, 2001). We also assume that the minimum thickness of the crown pillar is known. However, an important question remains unanswered: What is “the optimal transition depth” (Bakhtavar et al., 2008); that is, at which depth should the underground mining phase begin?

This paper answers this question. More specifically, the algorithm is tested on two different case studies, and the results of these tests are presented.

Related Work

As mentioned earlier, the transition problem arises when considering underground extraction as a means to extend the life of the mine considering the ore remaining in the deposit, and a few attempts have been made to solve it. Bakhtavar et al. (2008) proposed a sequential heuristic approach. First, the ultimate pit limits are determined based on block economic values and precedence constraints (Lerchs and Grossmann, 1965). Then, the underground mine boundaries are determined using a similar procedure and considering the crown pillar between the two mines (underground and open pit). The authors define the thickness of the optimal crown pillar as a function of geomechanical parameters such as span ratio, rock mass ratio, and rock mass characteristics that should be considered to avoid a collapse (Carter et al., 1998).

Bakhtavar et al. (2012) proposed an integer-programming model to determine open pit limits and underground boundaries simultaneously. The angle of the slope that guarantees stability is considered as a parameter of the model, and constraints are included to ensure that this slope angle is respected and that the crown pillar achieves the minimum thickness.

Chen et al. (2013) noted that there are two different deposits where the transition method could apply: those that have a horizontal extension and those that have a vertical extension. Depending on the type, the problem will be subject to subsidence or to pit wall collapse.

PROBLEM STATEMENT AND MATHEMATICAL MODEL

General Description

This paper aims to determine simultaneously the optimal economic envelope when both open pit and underground methods are used. We study the particular case where the panel/block caving method is used as an underground option. We think that this method is appropriate because it allows maintaining the production rates of the open pit operation.

Valuation of Blocks

Each block is assigned two different economic values: the open pit value and the underground value. The open pit value corresponds to the best value between extracting and processing the block versus extracting the block and not processing it (waste). The underground value corresponds to the case of extracting and processing the cost. Underground costs of blocks are higher because of the method and also the construction of an access to the level of extraction.

Crown Pillar

We define the crown pillar as being the minimum distance between the bottom of the open pit and the top of the underground mine that ensures minimum safety requirements and stability

Table 1. Parameters for crown pillar thickness

Parameter	C [Kg/ms^2]	S [m]	h [m]	RMR	γ_γ [Kg/m^{2}s^2]
Value	0.75	180	400	50	2.7

Table 2. Parameters to dilution calculation

Parameter	H_c [m]	HIZ [m]	s	dcf	RMR
Value	350	100	1.12	0.6	50

conditions. According to Backtavar et al. (2010), the thickness of the crown pillar (t) can be computed using the following formula:

$$t = \frac{13.22 \cdot C^{0.33} \cdot S^{0.41} \cdot h^{0.56}}{\gamma_\gamma^{\ 0.03} \cdot RMR^{0.66}}$$

where C is the cohesive strength, S is the stope span, h is the stope height, γ_γ is the specific weight of rock mass, and RMR is the Rock Mass Rating. In our case, the parameters are presented in Table 1.

Dilution

We used Laubscher's method (Laubscher 1994) to estimate the grade dilution in the underground mine. According to the author, the point of dilution entry is:

$$PDE(\%) = \frac{(H_c \cdot s - HIZ)}{H_c \cdot s} \cdot dcf \cdot 100$$

where H_c is the column height, s is the swallow factor, HIZ is the hight of interaction zone and dcf is the standard deviation factor. The parameters used in this paper are shown in Table 2.

Mathematical Formulation

Notation

We denote the set of blocks by $\mathbf{B} = \{1,2,\ldots,N\}$. Each block $i \in \mathbf{B}$ has an open pit value $p(i)$ and an underground value $q(i)$. $k(i)$ is the level of block i (its vertical coordinate, an integer). Levels 1 is at the bottom and L at the top.

Extraction through the pit is constrained by the slope of the pit walls modeled using *precedence arcs*. An arc is a pair $(i,j) \in \boldsymbol{B} \times \boldsymbol{B}$, representing the fact that block j has to be extracted before block i in order to gain access. We denote by $\boldsymbol{P}$ the set of all precedence arcs.

Extraction using underground is modelled similarly. For each block i there is a certain set of blocks that have to be extracted before it, but in this case these blocks are located below block i. We consider an arc set $\boldsymbol{Q} \subset \boldsymbol{B} \times \boldsymbol{B}$ encoding this.

Access to level k for the underground mine costs $R(k)$. Finally, the crown pillar consists of K levels (measured in blocks) and the maximum height of an underground extraction column is H, measured in block levels and not meters as H_c.

Mathematical Model

The main variables of the model correspond to the decision of extracting blocks using the open pit method or the underground one (see Figure 1 for an illustration). We define:

$x_i^P = 1$ if block i is extracted by the open pit method and 0 otherwise

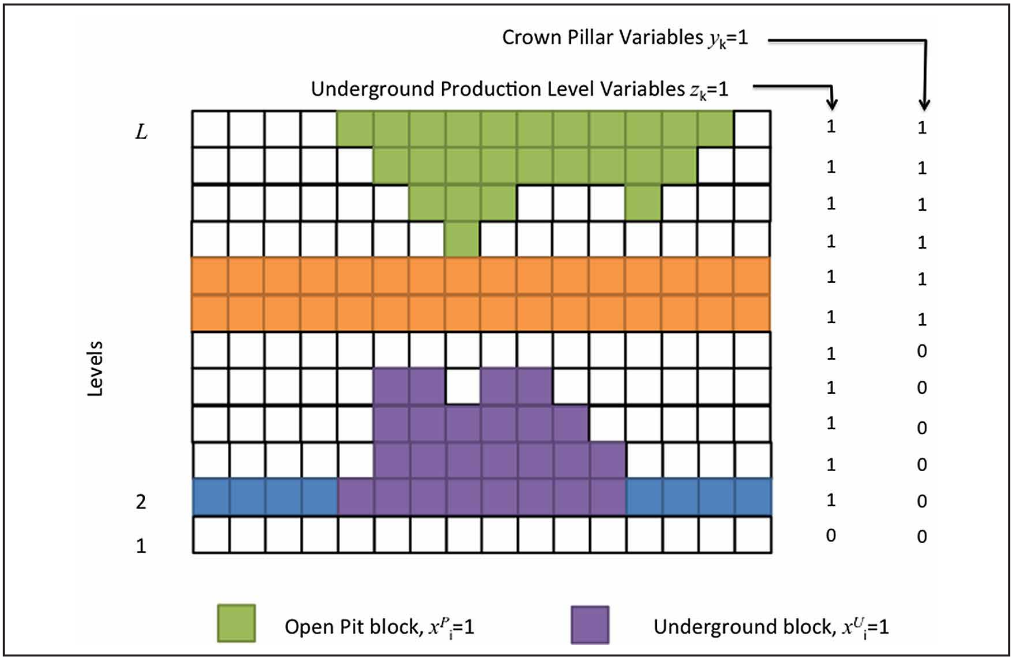

Figure 1. Problem variable definitions

x_i^U = 1 if block i is extracted by the underground method and 0 otherwise

Variables to locate the crown pillar and the production level of the underground mine are also required. For this, we define:

y_k = 1 if the bottom of the crown pillar is at level k or above
z_k = 1 if the bottom of the underground production level is at level k or above

Objective function	$\max V = \sum_{i \in \mathbf{B}} p(i)x_i^P + q(i)x_i^U - R(1)y_1 - \sum_{k=2}^{L} R(k)(y_k - y_{k-1})$	
At most one crown pillar	$y_k \geq y_{k-1}$	$\forall k = 2, \ldots, L$
At most one underground production level	$z_k \geq z_{k-1}$	$\forall k = 2, \ldots, L$
Pit Precedences	$x_i^P \leq x_j^P$	$\forall (i,j) \in \boldsymbol{P}$
Underground Precedences	$x_i^P \leq x_j^P + z_k - z_{k-1}$	$\forall (i,j) \in \boldsymbol{Q},\ k(i) > 1$
	$x_i^P \leq z_1$	$\forall (i,j) \in \boldsymbol{Q},\ k(i) = 1$
Open pit only above the crown pillar	$x_i^P \leq z_{k(i)-K}$	$\forall i,\ k(i) > K$
No underground above Crown pillar	$x_i^U + y_k \leq 1$	$\forall k, \forall i,\ k(i) \geq k$

Maximum height of underground mine	$x_i^U + z_1 \leq 1$ $x_i^U + z_k - z_{k-1} \leq 1$	$\forall\ k > 1, \forall i \in B,\ k(i) > k + H$

DESCRIPTION OF THE ALGORITHM

To solve the model presented in the previous section, we propose to parameterize the problem in terms of the crown pillar location and the production level location, so the optimal pit and the underground mine can be computed independently simply by using ultimate pits computations which are fast and do not miss the optimal solution.

Let us call k_C the level of the bottom of the crown pillar and k_P the level of the production level of the underground mine. If $k_C = 0$, $k_P = 0$ corresponds to the pure open pit case and $k_C = 0$, $k_P \neq 0$ the pure underground case, the possible combinations of (k_C, k_P) are

$$X = \begin{array}{c} \{(0,0)\} \cup \\ \{(0,k): k = 1,\ldots,L-H\} \cup \\ \{(k_C,k_P): k_C = 1,\ldots,L,\ \ k_P = 1,\ldots k_C - H\} \end{array}$$

In order to describe the algorithm, we also denote $B[k_1,k_1] = \{i \in B: k_1 \leq k(i) \leq k_2\}$, the set of blocks in levels k_1 to k_2.

Algorithm

1. Let P^* be the ultimate pit computed on the whole block model, v^* be its value, and $U^* = \emptyset$ be the current underground mine
2. For each $(k_C,k_P) \in X$:
   a. Compute ultimate pit in $B[k_C + K, L]$. Let v be its value.
   b. Compute best underground mine in $B[k_p, k_p + H]$. Let w be its value.
   c. If $v + w - R(k_P) > v^*$:
      i. Set $v^* = v + w - R(k_P)$
      ii. Set P^* to the pit obtained in 2.a.
      iii. Set U^* to the underground envelope obtained in 2.b.
3. Return P^*, U^*.

CASE STUDIES

In this section we briefly present the two case studies we used to test the model and the solution method described in the previous sections.

Gold Deposit

This block model has 82,932 blocks of size 10×10×10 [m^3] and a mean gold grade of 0.104 [ppm]. The deposit covers an area of 570×780 [m^2] and has a vertical extension of 570 [m] starting at level –295. Some general information on the resources is presented in Figure 2. The parameters used to compute the blocks' economic values are shown in Table 3.

For the underground mine, considerations like the vertical mining rate and the maximum high of the extraction column have been taken into account. The values of the parameters used are presented in Table 4.

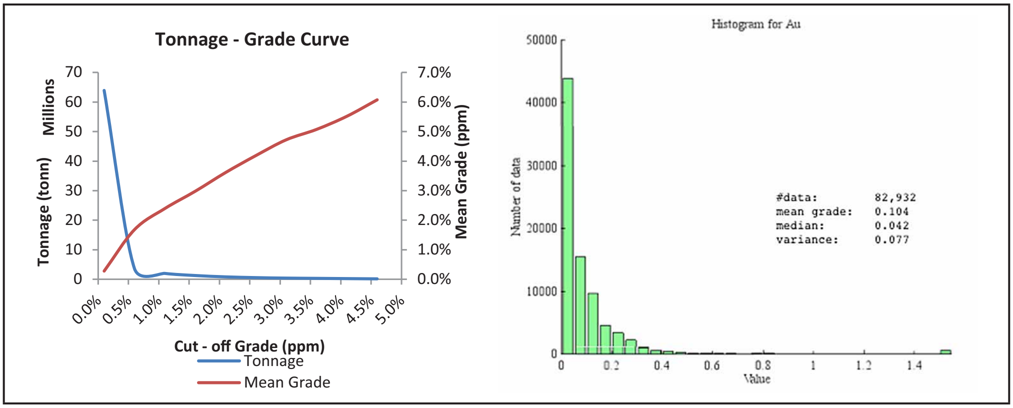

Figure 2. Tonnage-grade curves and grade distribution for gold deposit case study

Table 3. Cost and price parameters for the gold deposit

Open Pit & Underground	
Selling price	19.29 (US$/unit)
Selling cost	N/A
Mining cost	1.8 (US$/ton)
Proc cost—oxide ore	8.195 (US$/ton)
Recovery	90%

Table 4. Parameters for the underground caving mine

Underground Parameters	
Max column high	350 [m]
Area (grid 15×15 [m])	225 [m^2]
Vertical mining rate	66 [m/year]
Development cost	3000 [US$/$m^2$]
Height of Interaction zone	100 [100]

Copper Deposit

This deposit has 2,399,999 blocks of size 10×10×10 [m^3], with a mean copper grade of 0.584 [%]. The deposit covers an area of 1550×990 [m^2] and has a vertical extension of 1490 [m] starting at level 2755. General information on the resources is displayed in Figure 3. Cost parameters and price are listed inTable 5.

Underground parameters are the same as in the gold deposit, except that the maximum column height is 350 in this case.

RESULTS AND ANALYSIS

Gold Deposit

In this case, the optimal pit limit reaches a value over MUS$350 while the underground eraction through block/panel caving is economically unfeasible. This is reasonable because gd is usually found disseminated in the ore body (like in veins), therefore its extraction is not well suited for massive underground caving methods.

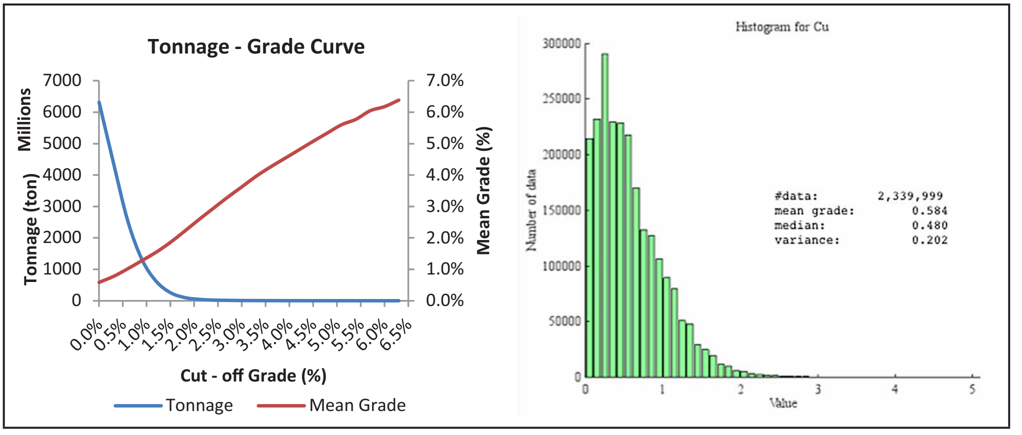

Figure 3. Tonnage-grade curve and histogram of the copper grade distribution for copper case study

Table 5. Cost and price parameters for the copper deposit

Open Pit & Underground	
Selling price	3 (US$/lb)
Selling cost	0.35 (US$/lb)
Mining cost	1.8 (US$/ton)
Proc cost—Oxide Ore	10 (US$/ton)
Recovery	90%

The bottom of the optimal pit can be set at the level 75 of the deposit; however, under this level there is no massive underground method of extraction.

Copper Deposit

In the case of the copper deposit, the value decomposed by mining technology and the total value are presented in Figure 5 and Table 6. As expected, the value of the underground mine increases with the height of the crown pillar, as the development investment required to access the orebody decreases, but this increase does not pay off for the loss in value of having a smaller pit. Therefore, the optimum is to have a pit as deep as possible (level 3850) and then an underground mine at level 3400. It is also worth noting that the best underground value (without considering development cost) is at level 3650.

CONCLUSIONS

We have proposed a mathematical model and a solution method to simultaneously determine the optimal open pit and underground envelopes, and thus the optimal transition depth. We have considered the particular case where the block caving method is used in the underground section. We have tested the solution method in two case studies, a gold deposit and a copper deposit. In the first case, as expected, the underground mine was not feasible for the underground method considered and therefore a more selective method should be tested. In the case of the copper deposit, the algorithm finds the best solution.

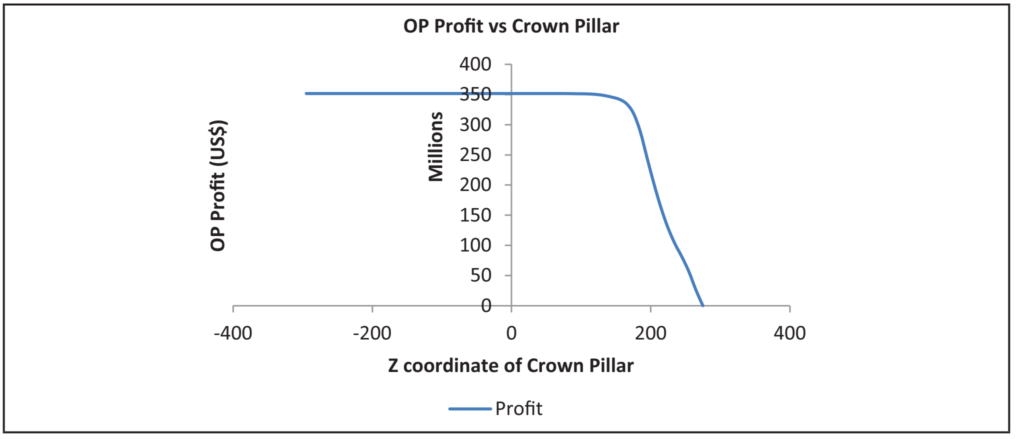

Figure 4. Profit curve of the gold deposit

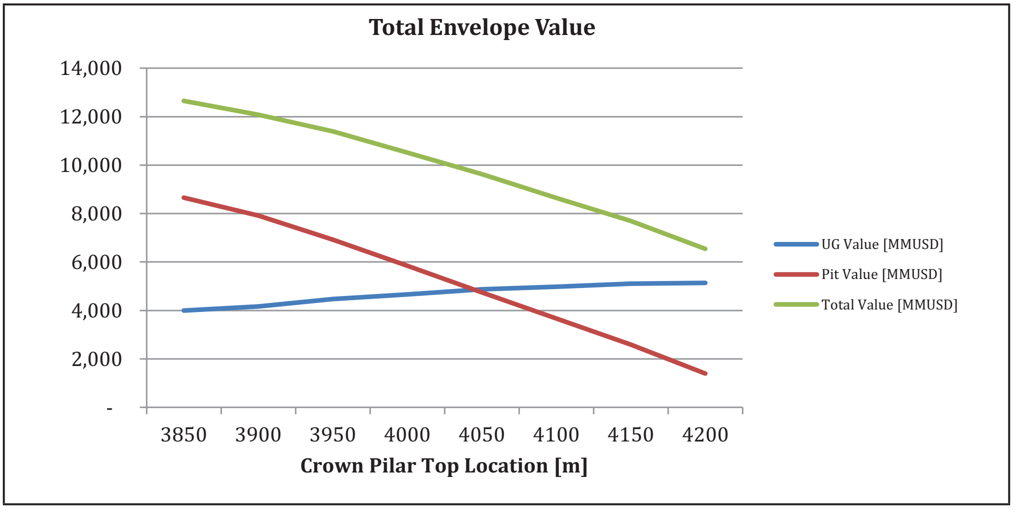

Figure 5. Envelope value depending on Crown pillar location

Table 6. Underground, open pit and total envelope value

CP Top Location [m]	Pit Value [MMUSD]	UG Foot Print Level [m]	UG Value w/o Development [MMUSD]	UG Value [MMUSD]	Total Value [MMUSD]
3850	8,658	3400	4,838	3,993	12,651
3900	7,918	3450	4,959	4,164	12,083
3950	6,921	3500	5,213	4,468	11,389
4000	5,849	3550	5,359	4,664	10,512
4050	4,746	3600	5,524	4,879	9,624
4100	3,666	3650	5,570	4,975	8,642
4150	2,594	3700	5,649	5,104	7,699
4200	1,399	3750	5,637	5,142	6,542

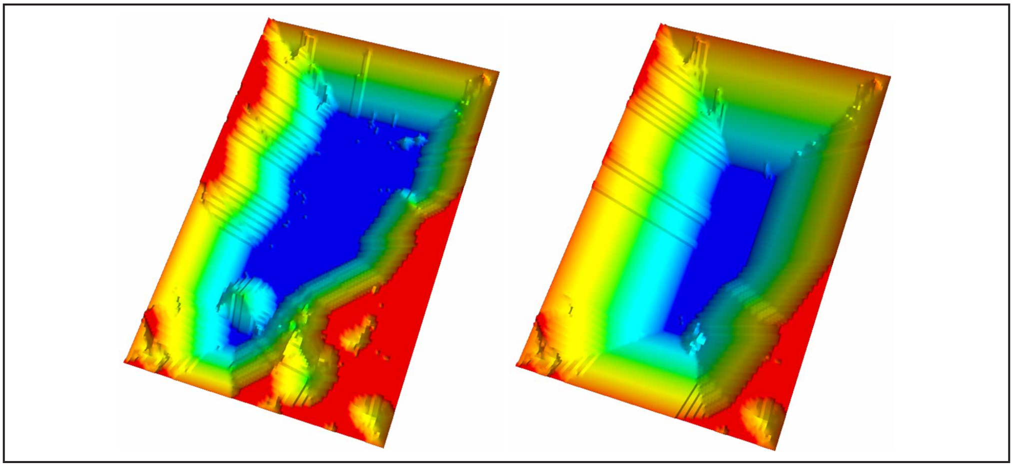

Figure 6. Final pit profiles. Left: Pit maximum depth is 4000[m]. Right: Pit maximum depth of 3850[m].

The proposed method is very fast, which encourages using it in more complex scenarios like studying the robustness of the joint envelope under geological uncertainty (that is grade variability, for example). Other potential research lines include scheduling of production and sensibility to prices.

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